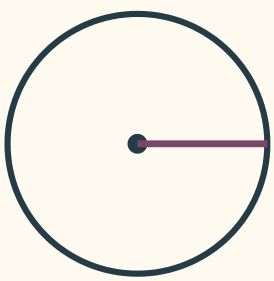
MATHS FIELD GUIDE · GRADE 9

Why circumference and area formulae work

OPENING PROBLEM

Find both circumference and area.



r = 4 cm

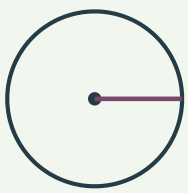
02 Read a circle from its radius
03 Rules and worked cases
04 π is exact; its decimal approximations are not
05 From the givens to the answer
06 Match the fraction of a turn
07 Check your reasoning
08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Read a circle from its radius

Circumference is the distance around. Area measures the region inside.

1

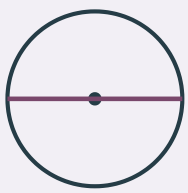


r = 4 cm

Find both circumference and area.

C = 8π cm and A = 16π cm². Notice the different units.

2



d = 10 cm

What error follows from treating diameter as radius?

Actual area is 25π cm². Using 10 as radius gives 100π, four times too large.

3

Radius	r	2r
Boundary	2πr	4πr
Area	πr²	4πr²

Which quantity doubles, and which quadruples?

Boundary scales with one length. Area scales in two directions.

4

Outer radius 5

↓

Inner radius 3

↓

Area = (25 − 9)π

How much area does a circular ring cover?

16π square units. Subtract the disks' areas, not their radii before squaring.

Radius 1
circumference 2π, area π.

Radius 2
circumference 4π, area 4π.

CHAPTER PROBLEM

A circle has radius 6 cm and circumference kπ cm. Find k.

12. C = 2πr = 12π, so k = 12.

MATHS REFERENCE · GRADE 9

Why circumference and area formulae work

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Circumference	For diameter 14, C = πd = 14π. The length-to-diameter ratio stays π across circles.
Area	Rearrange narrow sectors alternately. The limiting base is half the circumference, πr, and height r gives πr².
Double the radius	Circumference doubles but area quadruples. Radius 2 to 4 takes area from 4π to 16π.

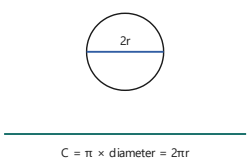
Read the representations

Every circle has the same circumference-to-diameter ratio

Enlarging a circle by scale factor k multiplies every boundary length and its diameter by k. Their ratio C/d is unchanged.

This common ratio is called π. Therefore C = πd = 2πr. Unrolling the rim preserves its boundary length.

The radius r is half the diameter, so C = πr would be missing a factor of two.

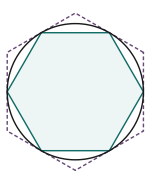


Polygon bounds explain increasingly accurate estimates

An inscribed polygon has a shorter perimeter than the circle's circumference. A circumscribed polygon has a longer one.

Increasing the number of sides tightens these lower and upper bounds. This provides a way to estimate the common ratio without claiming a finite polygon is the circle.

Historically, Archimedes obtained 223/71 < π < 22/7. Both endpoints are rational bounds, not exact values of π.





r = 3, h = 5

Find the dimensions of its unrolled wall.

Height 5 and width 6π. The curved area is 30π square units.

THEOREMS AND REPRESENTATIONS

π is exact; its decimal approximations are not

π is exact; its decimal approximations are not

π is irrational: it cannot equal a fraction of integers. Its decimal expansion neither terminates nor eventually repeats. The proof of this fact needs mathematics beyond this course.

Aryabhata used 62832/20000 = 3.1416, and Zu Chongzhi gave 355/113 ≈ 3.1415929. These are historical approximations to the same ratio.

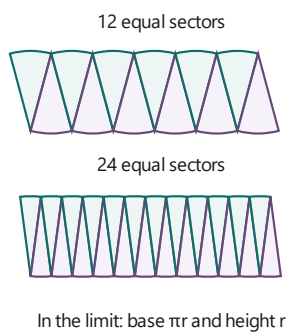
Keep π in an exact answer. Replace it by 3.14 or 22/7 only when a numerical estimate or a stated approximation is required.

Rearrange sectors to see where πr² comes from

Cut the circle into equal sectors and alternate their directions. Rearranging the pieces preserves the total area.

With finitely many sectors the outer edges are scalloped. As the sectors become narrower, the arrangement approaches a rectangle.

Its base approaches half the circumference, πr, and its height approaches r. The limiting area is therefore πr × r = πr².



Check the units while unrolling a circle

A wheel 0.7 m in diameter travels 0.7π m per complete turn without slipping. Ten turns cover 7π m, about 22 m.

Circumference is a length. The area π × 0.35² is measured in square metres and cannot give travel distance.



A cone has base radius 3 and slant height 9. Why is the sector angle 120°?

A full radius-9 circle has circumference 18π. The rim needs 6π, or one third of a turn.

WORKED EXAMPLES

From the givens to the answer

Keep an exact value and an estimate separate

1. A disk has radius 7 cm. Its exact circumference is 14π cm and its exact area is 49π cm².
2. Using $\pi \approx 22/7$ gives circumference ≈ 44 cm and area ≈ 154 cm². These values are estimates even though the arithmetic cancels neatly.
3. If the radius doubles, circumference doubles while area becomes four times as large.

Does 22/7 define π?

A learner writes $\pi = 22/7$ because the fraction is used in a calculation.

1. 22/7 is rational, whereas π is irrational, so they cannot be equal.
2. Write $\pi \approx 22/7$ and keep the approximation sign for the resulting measured quantity.
3. An exact symbolic expression such as 18π remains exact without choosing a decimal approximation.

Check another case

Example 1

Why should an answer using 22/7 in place of π be labelled approximate?

22/7 is a rational approximation and is not equal to π.

Substitution therefore changes the exact symbolic value.

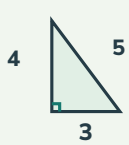
Use an approximation sign or describe the result as an estimate, even if the arithmetic gives an integer.

Example 2

A circle has radius 11 cm and circumference $k\pi$ cm. Find k.

22

$C = 2\pi \times 11 = 22\pi$.



A cone has radius 3 and vertical height 4. Which measure is the sector radius?

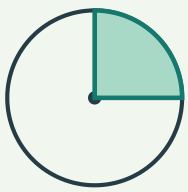
The slant height 5. It runs along the surface from tip to rim.

ILLUSTRATED PROBLEM SHEET

Match the fraction of a turn

For a sector of angle θ in degrees, multiply either the full circumference or full area by θ/360.

1

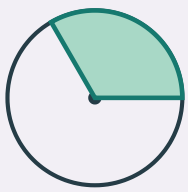


90°

For radius 8 cm, find the quarter arc.

A quarter of 16π cm is 4π cm.

2



120°

For radius 6 cm, find the sector area.

120/360 is one third. A third of 36π is 12π cm².

3



270°

What if the major sector is shaded?

Use the full 270/360 = 3/4. The smaller 90° angle refers to the unshaded part.

4



90°

Find the minor segment area for radius 4 cm.

Quarter disk 4π minus right triangle area 8 gives (4π − 8) cm².

60° sector
one sixth of the disk.

300° sector
five sixths of the disk.

CHAPTER PROBLEM

A disk has radius 8 cm and area kπ cm². Find k.

64. Area = π × 8² = 64π.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 A wheel has diameter 0.7 m and rolls ten turns without slipping. How far does it go?	Ten circumferences give 7π metres, close to 22 metres.
02 Does a sector's perimeter equal its arc length?	Include its two straight radii for a non-full sector. Arc length alone counts only the curved edge.
03 A circle has diameter 19 cm and circumference kπ cm. Find k.	19 $C = \pi d = 19\pi$.
04 A disk has diameter 18 cm and area kπ cm². Find k.	81 The radius is 9, so area = 81π.
05 Using π ≈ 3.14, estimate the circumference of a circle with radius 5 cm, in cm.	31.4 $C \approx 2 \times 3.14 \times 5 = 31.4$ cm.
06 Using π ≈ 22/7, estimate the area of a disk with radius 14 cm, in cm².	616 $A \approx (22/7) \times 196 = 616$ cm².

Compare a ring with a disk

A ring has outer radius 5 cm and inner radius 3 cm. Subtract areas to get $(25 - 9)\pi = 16\pi$ cm².

A solid disk of radius 4 cm has that same area. Its boundary is 8π cm long, while both edges of the ring total 16π cm.

Curved area = π × 3 × 5
↓
Base area = 9π
↓
Total = 24π

What changes if the cone is open at the base?

Only the 15π curved surface remains.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 A disk's radius is multiplied by 3. By what factor is its area multiplied?	9 Area depends on r^2, so the factor is $3^2 = 9$.
08 Which statement is correct? A) π equals 22/7 exactly B) π has a terminating decimal C) π changes when a circle gets larger D) π is irrational and 3.14 is an approximation	π is irrational and 3.14 is an approximation The same irrational ratio applies to every circle.
09 Explain the area derivation by rearranging equal sectors.	Alternating sectors preserves the sum of the piece areas. As the sector count grows, the scalloped arrangement approaches a rectangle of base half the circumference and height r. The limiting area is $\pi r \times r = \pi r^2$; a finite arrangement is not already an exact rectangle.
10 Why does scaling a circle preserve C/d?	The same enlargement multiplies circumference by k. It also multiplies diameter by k. The common scale factor cancels in kC/(kd), leaving the original ratio.
11 A disk's radius is halved. Its area becomes what fraction of the original?	1/4 The area factor is $(1/2)^2 = 1/4$.

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