MATHS FIELD GUIDE · GRADE 9

Why algebraic square identities work

OPENING PROBLEM

Where does 2ab in $(a + b)^2$ come from?

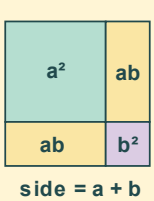
×	a	b
a	a^2	ab
b	ab	b^2

02 Rules and worked cases
03 Match each pattern to its expansion
04 Recognise the conditions
05 From the givens to the answer
06 Split a rectangle to see the distributive law
07 Check your reasoning
08 Check your reasoning

ALGEBRAIC IDENTITIES · FORMULA REFERENCE

Why algebraic square identities work

Recognise the form	Use the identity	See it in numbers or symbols
Square of a sum	$(a + b)^2 = a^2 + 2ab + b^2$	$(3x + 2)^2 = 9x^2 + 12x + 4$
Square of a difference	$(a - b)^2 = a^2 - 2ab + b^2$	$(5y - 1)^2 = 25y^2 - 10y + 1$
Difference of squares	$a^2 - b^2 = (a - b)(a + b)$	$49t^2 - 4 = (7t - 2)(7t + 2)$
Two linear factors	$(x + a)(x + b) = x^2 + (a + b)x + ab$	$(x - 3)(x + 8) = x^2 + 5x - 24$
Cube of a sum	$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$	$(x + 2)^3 = x^3 + 6x^2 + 12x + 8$
Cube of a difference	$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$	$(2x - 1)^3 = 8x^3 - 12x^2 + 6x - 1$
Sum of cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$	$8x^3 + 27 = (2x + 3)(4x^2 - 6x + 9)$
Difference of cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	$27y^3 - 1 = (3y - 1)(9y^2 + 3y + 1)$
Three-term square	$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$	$(x + y + 1)^2 = x^2 + y^2 + 1 + 2xy + 2y + 2x$
Group a common pair	$ax + ay + bx + by = (a + b)(x + y)$	$mn + 2m + 3n + 6 = (m + 3)(n + 2)$



Where does the 2ab come from?

The top-right and bottom-left rectangles each have area ab. Together they supply 2ab. As a check, when a = b = 3, the large square has area 36. The two small squares alone give only 18.

ILLUSTRATED PROBLEM SHEET

Match each pattern to its expansion

Multiplying two sums forms every pairwise product.

1

×	a	b
a	a^2	ab
b	ab	b^2

Where does 2ab in $(a + b)^2$ come from?

The two rectangular off-diagonal products are both ab.

2

$$(3x - 2)^2$$
$$\downarrow$$
$$(3x)^2 - 2(3x)(2) + 2^2$$
$$\downarrow$$
$$9x^2 - 12x + 4$$

Which signs change in a squared difference?

The middle product is negative. The constant is positive because $(-2)^2 = 4$.

3

$$49x^2 - 16$$
$$\downarrow$$
$$(7x)^2 - 4^2$$
$$\downarrow$$
$$(7x - 4)(7x + 4)$$

Why is there no middle term?

The cross-products +28x and −28x cancel.

4

×	x	−3
x	x^2	−3x
5	5x	−15

Collect the products of $(x + 5)(x - 3)$.

The middle terms combine to 2x, giving $x^2 + 2x - 15$.

$(p + q)^2 - (p - q)^2$
4pq.

$(p + q)^2 + (p - q)^2$
$2p^2 + 2q^2$.

CHAPTER PROBLEM

Find the coefficient of x in the expansion of $(x + 6)^2$.

12. The mixed term is $2 \times x \times 6 = 12x$.

RULES AND EXAMPLES

Recognise the conditions

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Square of a sum	$(2x + 3)^2 = (2x)^2 + 2(2x)(3) + 3^2 = 4x^2 + 12x + 9$.
Square of a difference	$(3y - 2)^2 = 9y^2 - 12y + 4$. Only the mixed term changes sign.
Conjugate factors	$(5t + 1)(5t - 1) = 25t^2 - 1$. The two linear terms cancel.

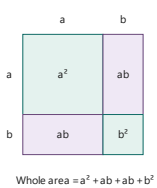
Read the representations

A larger square contains four regions

Split each side of a square of length a + b into lengths a and b.

The four regions have areas a^2, ab, ab and b^2. Their total is the area of the whole square.

Therefore $(a + b)^2 = a^2 + 2ab + b^2$. The two rectangles explain the coefficient 2.



The geometric model uses positive lengths. Expansion verifies the identity for all real a and b.

Subtraction changes the mixed term

Replace b by −b in the addition identity.

$(a - b)^2 = a^2 - 2ab + b^2$. The last term stays positive because $(-b)^2 = b^2$.

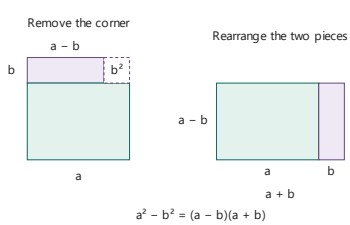
The middle term changes sign, rather than every term changing sign.

A difference of squares rearranges into a rectangle

For a > b > 0, remove a b by b square from an a by a square.

The remaining area can be rearranged into a rectangle with sides a − b and a + b.

Thus $a^2 - b^2 = (a - b)(a + b)$. Expansion confirms the identity beyond the positive-length picture.



Two pieces retain their areas when rearranged. The combined rectangle has the same area as the square with a corner removed.

$$(a + b)^3$$
$$\downarrow$$
$$(a^2 + 2ab + b^2)(a + b)$$
$$\downarrow$$
$$a^3 + 3a^2b + 3ab^2 + b^3$$

Why are the middle coefficients three?

The b can come from any one of three brackets, or the a can come from any one bracket.

WORKED EXAMPLES

From the givens to the answer

Does squaring distribute over addition?

A learner writes $(x + 4)^2 = x^2 + 16$.

1. Squaring means multiplying $(x + 4)(x + 4)$.
2. Distribution produces $x^2 + 4x + 4x + 16$.
3. The two mixed terms are essential: $x^2 + 8x + 16$.

Calculate 98^2 using a nearby whole hundred

1. Write $98 = 100 - 2$.
2. $98^2 = 10000 - 400 + 4$.
3. The result is 9604. A value slightly below 10000 is sensible.

Factor $9t^2 - 25$

1. Recognise $(3t)^2 - 5^2$.
2. Apply $a^2 - b^2 = (a - b)(a + b)$.
3. The result is $(3t - 5)(3t + 5)$. Multiplication gives $9t^2 - 25$ again.

Check another case

Example 1

Which factors $25 - q^2$?

$(5 - q)^2 \cdot (25 - q)(1 + q) \cdot (5 - q)(5 + q) \cdot (5 + q)^2$

$(5 - q)(5 + q)$

It is $5^2 - q^2$.

Example 2

Why does an area proof use positive lengths while the identity works for negative real values too?

Physical side lengths in the picture are positive.

Distribution proves the algebraic equality without that geometric restriction.

The picture explains the structure, and expansion verifies the full real-number identity.

$$(2x - 1)^3 \downarrow 8x^3 - 12x^2 + 6x - 1$$

Which signs alternate?

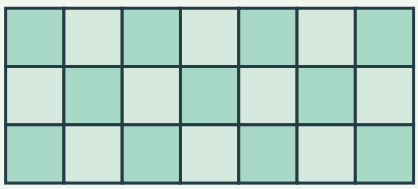
Terms with an odd number of negative ones are negative. At $x = 1$ the expansion gives 1.

ILLUSTRATED PROBLEM SHEET

Split a rectangle to see the distributive law

One common height multiplies every piece of the width.

1



Split this 3 by 7 array after five columns. What equality appears?

$3 \times 7 = 3 \times 5 + 3 \times 2$. The two pieces cover exactly the original rectangle.

2

$$4(x + 3)$$
$$\downarrow$$
$$4x + 4 \times 3$$
$$\downarrow$$
$$4x + 12$$

Which term is often missed?

The 3 is also multiplied by 4. At $x = 2$ both sides give 20.

3

$$5x + 15$$
$$\downarrow$$
$$5(x + 3)$$

Can you reverse the split?

Both terms contain 5. Dividing each by 5 leaves x and 3 inside the bracket.

4

$$-2(x - 4)$$
$$\downarrow$$
$$-2x + 8$$

Why is the constant positive?

It comes from multiplying two negative factors. Test $x = 0$: both sides give 8.

$2(x + y)$
$2x + 2y$.

$2x + y$
only x has coefficient 2.

CHAPTER PROBLEM

Calculate 102^2 using a square identity.

10404. $(100 + 2)^2 = 10000 + 400 + 4$.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Use one identity to calculate 99^2 without long multiplication.	$(100 - 1)^2 = 10{,}000 - 200 + 1 = 9801$.
02 Which factors $x^2 - 49$? A) $(x - 7)^2$ B) $(x - 7)(x + 7)$ C) $(x - 49)(x + 1)$ D) $(x + 7)^2$	$(x - 7)(x + 7)$ $x^2 - 7^2$ is a difference of squares.
03 Which is the expansion of $(y - 3)^2$? A) $y^2 - 9$ B) $y^2 - 6y + 9$ C) $y^2 + 6y + 9$ D) $y^2 - 3y + 9$	$y^2 - 6y + 9$ Use $y^2 - 2 \times y \times 3 + 3^2$.
04 Is $3a + 2b$ always $5ab$?	No. With $a = b = 2$ the two sides are 10 and 20. Unlike terms cannot be merged that way.
05 Find the coefficient of p in $(2p + 5)^2$.	20 $2 \times 2p \times 5 = 20p$.
06 Which factors $16z^2 - 1$? A) $(4z - 1)(4z + 1)$ B) $(16z - 1)(z + 1)$ C) $(4z - 1)^2$ D) $(4z + 1)^2$	$(4z - 1)(4z + 1)$ The two squared quantities are 4z and 1.

$8x^3 + 27$
↓
$(2x)^3 + 3^3$
↓
$(2x + 3)(4x^2 - 6x + 9)$

How does the middle sign help?

When the factors multiply, both mixed-degree terms cancel.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Calculate 53 × 47 using a difference-of-squares identity.	2491 (50 + 3)(50 − 3) = 2500 − 9 = 2491.
08 Which expression is a perfect-square expansion? A) $x^2 + 10x + 25$ B) $x^2 + 10x + 10$ C) $x^2 + 25$ D) $x^2 - 25$	$x^2 + 10x + 25$ $x^2 + 2 \times x \times 5 + 5^2 = (x + 5)^2$.
09 Explain geometrically why $(a + b)^2$ contains 2ab.	Splitting both sides creates two distinct rectangles. Each rectangle has sides a and b and area ab. Both belong to the whole square, giving ab + ab = 2ab.
10 Explain why $a^2 + b^2$ is generally different from $(a + b)^2$.	The second expression also contains 2ab after expansion. That mixed term is usually non-zero. For example a = 1 and b = 2 give 5 and 9 respectively.
11 Find the coefficient of x in $(x - 8)^2$.	-16 The mixed term is $-2 \times x \times 8$.

$$a^3 - b^3 \downarrow (a - b)(a^2 + ab + b^2)$$

How can you check the signs quickly?

Multiply out. The mixed products cancel, leaving the two original cubes.

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