MATHS FIELD GUIDE · GRADE 9

Weighted averages and interpretation

OPENING PROBLEM

What is the combined mean?

Group	Count	Mean
A	4	80
B	16	60

02 Ask what one observation represents
03 Rules and worked cases
04 A weighted mean is a balance point
05 From the givens to the answer
06 Read centre and spread separately
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08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Ask what one observation represents

Combine totals and counts before dividing. A mean of group means weights groups equally.

1

Group	Count	Mean
A	4	80
B	16	60

What is the combined mean?

(320 + 960)/20 = 64. The larger group pulls the mean towards 60.

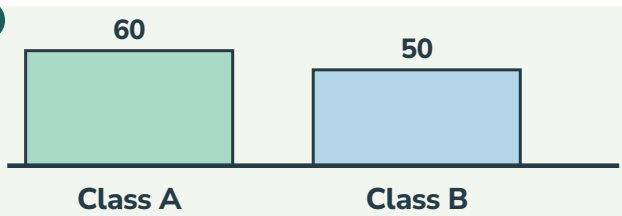
2

Value	2	5	8
Count	1	2	1

Find the mean without writing the full list.

Weighted total 2 + 10 + 8 = 20. Four observations give mean 5.

3



Bars show percent cycling. A has 20 pupils and B has 30. Which has more cyclists?

A has 12 cyclists and B has 15. A higher percentage need not mean a higher count.

4

Fresh sample: 20 pupils

↓

Only cycling-club members asked

Why might its travel survey overstate cycling?

The selection method favours cyclists. More responses from that same club do not remove the bias.

A group with zero observations
contributes zero weight.

All groups of equal size
group means receive equal weights.

CHAPTER PROBLEM

Five values have mean 12 and fifteen values have mean 20. Find the combined mean.

18. (5 × 12 + 15 × 20)/20 = 18.

MATHS REFERENCE · GRADE 9

Weighted averages and interpretation

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Combine two groups	Ten pupils average 60 and thirty average 80. Combined mean = (600 + 2,400)/40 = 75.
Recover a missing group	Overall 20 scores average 14; five average 20. Remaining fifteen average (280 – 100)/15 = 12.
Equal group sizes	The simple mean of the group means works when each mean has the same weight.

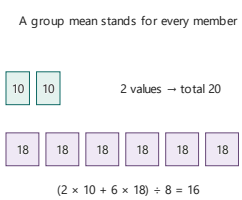
Read the representations

Unequal groups need unequal influence

A group mean stands for the observations in that group. Its contribution to the combined total is group size × group mean.

The combined mean is the sum of those contributions divided by the total number of observations.

The larger group in the picture supplies more observations and therefore more weight. Simply averaging the two group means would give the groups equal influence.



Weights tell how much each value contributes

For values x with weights w, weighted mean = ∑(wx)/∑w. Here ∑ means add over all the entries.

A frequency is one kind of weight. For frequency data, ∑f is the number of observations and ∑fx is the sum of their values.

Weights must be nonnegative and have positive total in this model. Weights are relative: multiplying every weight by the same positive factor preserves the mean.

Question	Population
School travel	all pupils
Club travel	club members

Who is missing from a cycling-club survey of school transport?

Pupils outside the club. They may use transport differently.

THEOREMS AND REPRESENTATIONS

A weighted mean is a balance point

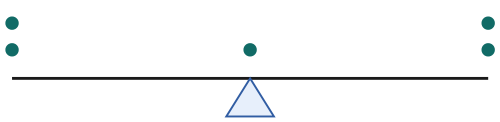
A weighted mean is a balance point

Place repeated equal weights at each numerical value. The mean balances the weighted distances to its left and right.

With nonnegative weights, the result lies between the least and greatest values carrying positive weight.

An answer outside that interval signals an error in the model or calculation. A balance drawing illustrates the idea; the weighted-sum calculation gives the exact result.

The mean balances weighted distances



$2(5 - 2) = 2(8 - 5)$

Total weight 5; weighted mean 5

Percentage weights must refer to compatible scores

A weighted course score can combine an assignment and an examination if their scores are first expressed on the required common scale.

For example, weights 40% and 60% mean $0.4x + 0.6y$ when x and y are percentage scores. Their weights sum to 1.

Do not average raw marks from different maximum scores without converting or following the specified assessment rule. State what the weights represent.

Can a smaller group move the overall mean?

Four pupils average 80 and sixteen average 60. Combine totals, 320 + 960 = 1280, then divide by 20 to get 64.

The overall mean sits four above 60 because the higher-scoring group makes up only one fifth of the pupils.

Group	Records	Cycle
A	20	8
B	10	6

Pool the observations. What fraction cycle?

14/30 = 7/15. Averaging 40% and 60% would give both groups equal weight.

WORKED EXAMPLES

From the givens to the answer

Pool group means by rebuilding totals

1. Group P has 12 learners with mean score 7. Group Q has 18 learners with mean score 9 on the same scale.
2. Their score totals are 84 and 162. The combined mean is (84 + 162)/(12 + 18) = 246/30 = 8.2.
3. The simple average 8 would give the smaller group the same influence as the larger one. It is valid here only if the group sizes are equal.

Can two success percentages always be averaged?

Two events report success rates of 60% and 90%. A learner claims that the combined rate must be 75%.

1. The combined rate depends on the number of trials in each event.
2. If the event sizes differ, recover their success counts and divide total successes by total trials.
3. Without sizes, 75% is justified only under an equal-size assumption, which the information does not supply.

Check another case

Example 1

Values 20 and 50 have weights 4 and 2. Find the weighted mean.

30

(80 + 100)/6 = 30.

Example 2

Two classes have different mean heights but give no class sizes. What else is needed to find the combined mean?

The number of observations represented by each class mean is needed.

Multiply each mean by its count to reconstruct the class total.

Divide the sum of these totals by the sum of the counts, using compatible height units.

Day	Monday	Tuesday
Rain?	yes	no
Walkers	8	20

Does the difference prove rain caused the change?

It is consistent with that idea, but other day-to-day differences may explain it.

ILLUSTRATED PROBLEM SHEET

Read centre and spread separately

The mean balances all values. The median uses position in sorted order.

1

Scores	2	4	9
Total	15		

What is the mean?

Redistribute fifteen points equally across three scores. Each gets five.

2

2 4 9

Which score is the median?

Four is in the middle of the ordered list. It differs from the mean of five.

3

1 3 3 9

Which score is most frequent?

Three occurs twice. It is also the median. The range is 9 – 1 = 8.

4

List	Low	High
A	4	6
B	1	9

The two-value lists both average five. Do they spread equally?

No. The ranges are two and eight.

Add 2 to every observation
mean rises by 2.

Add 2 to every observation
range is unchanged.

CHAPTER PROBLEM

Scores 60 and 80 have weights 1 and 3. Find their weighted mean.

75. (60 + 240)/4 = 75.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 What if both group sizes are equal?	Then the combined mean equals the simple average of the two group means.
02 Three scores average six. Two scores are four and nine. Find the third.	The total must be eighteen. Subtract thirteen to get five.
03 Value 4 occurs twice and value 10 occurs four times. Find the mean.	8 $(4 \times 2 + 10 \times 4)/6 = 8$.
04 Groups of 8 and 12 learners have mean scores 50 and 65. Find the pooled mean.	59 $(400 + 780)/20 = 59$.
05 Course percentages 70 and 90 carry weights 30% and 70%. Find the final percentage score; enter the number only.	84 $0.30 \times 70 + 0.70 \times 90 = 84$.
06 A dataset of 9 observations has mean 14. What is its total?	126 Total = count × mean = 9×14.

Measure	Before	After
Total pupils	20	40
Cyclists	10	16

Which rose: the headcount or share cycling?

The count rose by six. The share fell from 50% to 40%.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Six observations have total 42. Adding four observations with mean 12 gives what combined mean?	9 (42 + 48)/10 = 9.
08 Values 12 and 28 have positive weights. Which result could be their weighted mean? A) 10 B) 30 C) 40 D) 22	22 A positive-weight mean must lie between 12 and 28.
09 Why does doubling every weight preserve a weighted mean?	Every contribution wx doubles, so the numerator doubles. The total weight in the denominator also doubles. The common factor cancels, leaving the same ratio.
10 A group of 10 has mean 4 and a group of 30 has mean 8. Explain why the pooled mean is closer to 8.	The larger group supplies three times as many observations. Its total contribution is 30 × 8, compared with 10 × 4. The pooled mean is 280/40 = 7, which is closer to 8 than to 4.
11 Groups of 7 and 14 have means 6 and 9. Find their pooled mean.	8 (42 + 126)/21 = 8.

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