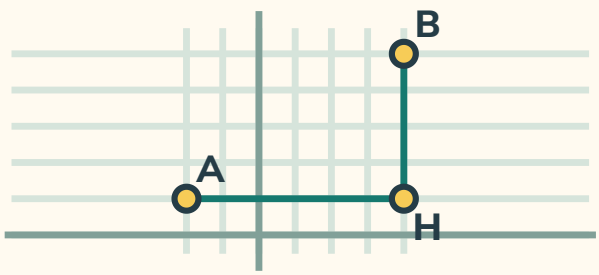
MATHS FIELD GUIDE · GRADE 9

Using Pythagoras and its converse

OPENING PROBLEM

Find the direct distance from A(−2, 1) to B(4, 5).



A (-2, 1)
H (4, 1)
B (4, 5)

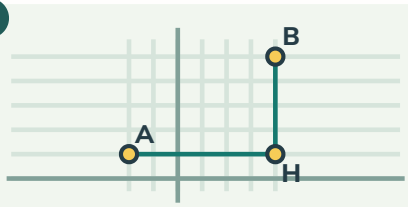
02 Build the right triangle between two points
03 Rules and worked cases
04 From the givens to the answer
05 Construct a length from perpendicular steps
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Build the right triangle between two points

The horizontal and vertical displacements are perpendicular. Their squares add to the squared direct distance.

1

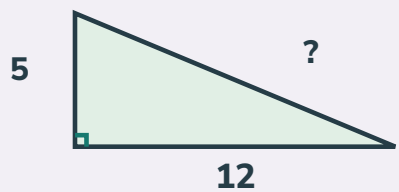


A (-2, 1)
H (4, 1)
B (4, 5)

Find the direct distance from A(−2, 1) to B(4, 5).

The legs are 6 and 4. AB = $\sqrt{52} = 2\sqrt{13}$. The broken route via H is 10 units.

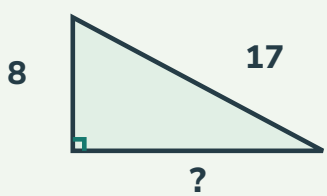
2



A right triangle has legs 5 and 12. How long is its hypotenuse?

$5^2 + 12^2 = 169$, so the longest side is 13.

3



The hypotenuse is 17 and one leg is 8. Recover the other leg.

Subtract squares: 289 − 64 = 225. The missing positive length is 15.

4

$$7^2 + 24^2$$
$$\downarrow$$
$$49 + 576 = 625$$
$$\downarrow$$
$$25^2$$

Can sides 7, 24 and 25 make a right triangle?

Yes. The converse places the right angle opposite the longest side, 25.

Coordinate changes (0,7)
distance 7.

Coordinate changes (−3,4)
distance 5.

CHAPTER PROBLEM

A right triangle has legs 9 cm and 12 cm. Find the hypotenuse in cm.

15. Square the legs and add: $9^2 + 12^2 = 225$. The hypotenuse is the positive square root.

MATHS REFERENCE · GRADE 9

Using Pythagoras and its converse

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Two legs known	Legs 8 and 15 give $c^2 = 64 + 225 = 289$, hence $c = 17$.
One leg missing	With $c = 13$ and $a = 12$, $b^2 = 169 - 144 = 25$, so $b = 5$.
Test for a right angle	For 8, 10, 13 compare $64 + 100$ with 169. They differ; the converse does not apply.

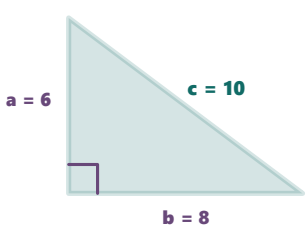
Read the representations

The right angle determines the sides

In a right triangle, the side opposite the 90° angle is the hypotenuse c. It is the longest side.

The other two sides are the legs a and b. Pythagoras states $a^2 + b^2 = c^2$.

Find the hypotenuse



$a^2 + b^2 = c^2$

$6^2 + 8^2 = 36 + 64 = 100$

$c = \sqrt{100}$

$c = 10$

Pythagoras theorem

The squares on the legs have total area equal to the square on the hypotenuse.

The theorem and its converse

Given a right angle, the theorem calculates or relates lengths.

Given three positive side lengths that form a triangle, compare the square of the longest side with the sum of the other squares. Equality proves a right angle.

First check that the two shorter sides add to more than the longest side. A sketch alone does not establish a right angle.

Converse of Pythagoras

If $a^2 + b^2 = c^2$, the angle opposite c is 90°

$5^2 + 12^2 = 25 + 144 = 169$

$13^2 = 169$

They are equal, so the triangle is right-angled at the join of a and b

This is the converse of Pythagoras

The converse tells you when the angle is right
Here $5^2 + 12^2 = 13^2$, so the triangle is right-angled.

Triangle	Side 1	Side 2	Side 3
ABC	3	4	5
PQR	3	4	5

Can all three matching side lengths establish congruence?

Yes, by SSS. Match the vertex opposite each corresponding side.

WORKED EXAMPLES

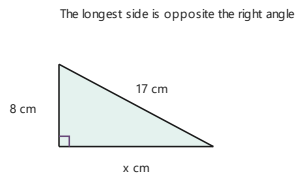
From the givens to the answer

Which side receives the sum?

A learner labels the vertical side c because it stands upright.

1. Locate the right-angle mark.
2. The opposite side is the hypotenuse, whatever the orientation.
3. Only its square equals the sum of the two leg squares.

Find a leg when c = 17 cm and one leg is 8 cm



The 17 cm side is opposite the marked right angle. The unknown is a leg.

1. Let the missing leg be x. Pythagoras gives $x^2 + 8^2 = 17^2$.
2. Subtract 64: $x^2 = 289 - 64 = 225$.
3. Since a length is positive, x = 15 cm.
4. Check: $8^2 + 15^2 = 64 + 225 = 289$.

Test a triangle with sides 6, 7 and 10

1. The longest side is 10. The triangle inequality holds: 6 + 7 > 10.
2. $6^2 + 7^2 = 36 + 49 = 85$.
3. $10^2 = 100$, which is not 85. Therefore it is not a right triangle.

Check another case

Example 1

Sides 10, 24 and 26 form a right triangle because...

$10 + 24 > 26$ · $10^2 + 24^2 = 26^2$ · $10^2 + 26^2 = 24^2$ · All lengths are even

$10^2 + 24^2 = 26^2$

$100 + 576 = 676 = 26^2$.

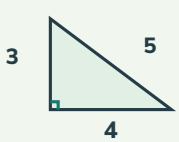
Example 2

Explain what you must know before using $a^2 + b^2 = c^2$ to calculate a side.

The triangle must be right-angled, stated or justified.

c must be opposite the right angle.

The known lengths and units must be consistent.



Two triangles each have legs 3 and 4 meeting at 90°. Is that enough?

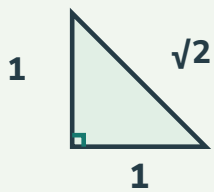
Yes. Two corresponding sides and their included right angle give SAS.

ILLUSTRATED PROBLEM SHEET

Construct a length from perpendicular steps

A new perpendicular leg gives a right triangle. Label the square of the new radius before taking the root.

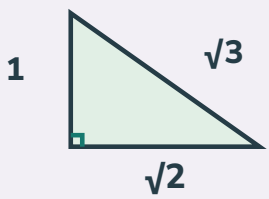
1



From a unit segment, add another unit at right angles. What is the diagonal?

Its square is 1 + 1 = 2, so its positive length is √2.

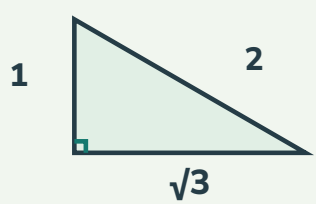
2



Build a unit leg perpendicular to the new radius. What comes next?

The squared radius becomes 2 + 1 = 3.

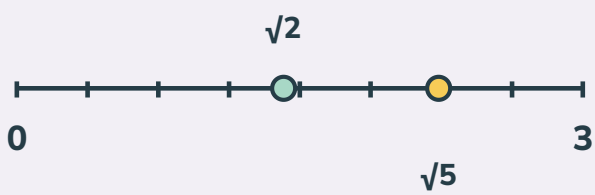
3



What happens on the next unit step?

The squared length is four. The radius is now the integer two.

4



How can a length become a point on a number line?

Set a compass to the constructed length. With centre zero, transfer it to the positive ray.

Radius squared 7, new perpendicular unit step
squared radius 8.

The length increase
√8 − √7, less than one.

CHAPTER PROBLEM

A right triangle has legs 5 m and 12 m. Find its hypotenuse in m.

13. Square the legs and add: $5^2 + 12^2 = 169$. The hypotenuse is the positive square root.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Is a route 3 units east and 4 north 7 units from its start?	The route length is 7. The straight-line distance is 5.
02 Which three lengths form a right triangle? A) 4, 5, 6 B) 7, 24, 25 C) 5, 6, 8 D) 3, 4, 6	7, 24, 25 $7^2 + 24^2 = 49 + 576 = 625 = 25^2$.
03 Could the added unit segment point in an arbitrary direction and give the same calculation?	No. The perpendicular condition is what allows addition of squares.
04 A right triangle has hypotenuse 10 cm and one leg 6 cm. Find the other leg in cm.	8 $x^2 = 100 - 36 = 64$, so $x = 8$.
05 Which comparison proves that a triangle with sides 8, 15 and 17 is right-angled? A) $8 + 15 = 17$ B) $8^2 + 15^2 = 17^2$ C) $8^2 + 17^2 = 15^2$ D) $8 \times 15 = 17^2$	$8^2 + 15^2 = 17^2$ The longest side is 17, and $64 + 225 = 289$.
06 A rectangle is 12 cm by 16 cm. Find its diagonal in cm.	20 The diagonal is the hypotenuse: $d^2 = 144 + 256 = 400$.

Equal angles: 40°, 60°, 80°
↓
Same shape
↓
Size still free

Do three equal angle pairs prove congruence?

They give similarity. One triangle may be an enlargement of the other.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Can sides 2 cm, 3 cm and 6 cm form a triangle? A) Yes, and it is right-angled B) Yes, but it is not right-angled C) No, because 2 + 3 < 6 D) No, because all three sides must be equal	No, because 2 + 3 < 6 The sum of the two shorter sides must exceed the longest side.
08 A 10 m ladder touches a vertical wall. Its foot is 6 m from the wall on level ground. Find the height in m.	8 The wall and ground give a right angle. $h^2 = 10^2 - 6^2 = 64$.
09 A triangle looks right-angled but has sides 9, 10 and 14. What can you conclude?	The lengths form a triangle. $9^2 + 10^2 = 181$, while $14^2 = 196$. Since the squares are unequal, the triangle is not right-angled.
10 Why do we choose the positive square root for a triangle side?	Squaring an equation can allow positive and negative algebraic solutions. A side length must be positive. The geometrical context excludes the negative solution.
11 Find a missing leg when the hypotenuse is 13 and the other leg is 12.	5 $x^2 = 169 - 144 = 25$, so $x = 5$.

Third side	3	7	11
Other sides	4 and 7	4 and 7	4 and 7
Triangle?	no	yes	no

Why do endpoints fail?

The third side must be strictly between 3 and 11. Equality collapses the triangle into a straight line.

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