MATHS FIELD GUIDE · GRADE 9

Triangle inequalities and shortest distance

OPENING PROBLEM

Why do these three lengths fail?

4 7 11

02 Test the boundary of a triangle
03 Rules and worked cases
04 The perpendicular is the shortest route to a line
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08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Test the boundary of a triangle

For two positive sides a and b, the third length x must satisfy $|a - b| < x < a + b$.

1

4 7 11

Why do these three lengths fail?

Four plus seven equals eleven. The three segments can only lie flat.

2

$a = 4, b = 7$

↓

$3 < x < 11$

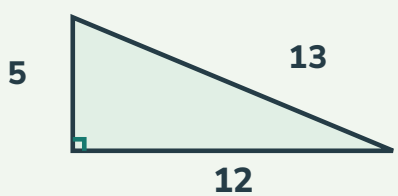
↓

$x = 4,5,6,7,8,9,10$

How many positive integer choices?

Seven. Neither endpoint is allowed.

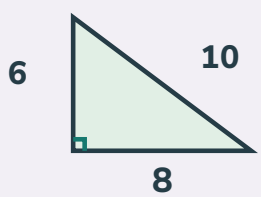
3



Which route between the ends of the hypotenuse is shorter?

The direct segment has length 13. The two-leg path measures 17.

4



From a point 6 m above a line, compare the perpendicular with a route landing 8 m sideways.

The perpendicular is 6 m. The sloping segment is 10 m, showing why the nearest point is the perpendicular foot.

Third side between 2 and 8
integers 3 through 7.

Third side exactly 8
fails the strict upper bound.

CHAPTER PROBLEM

P is 8 m from a line at perpendicular foot H. Q is 15 m from H on that line. Find PQ in m.

17. $PQ^2 = 8^2 + 15^2 = 289$, hence PQ = 17 m.

MATHS REFERENCE · GRADE 9

Triangle inequalities and shortest distance

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Test feasibility	Sides 4,7,12 fail since $4 + 7 < 12$.
Bound a missing side	With sides 5 and 8, the third length x satisfies $3 < x < 13$. Equality would flatten the triangle.
Shortest path to a line	If H is the perpendicular foot and Q another point on the line, $PQ^2 = PH^2 + HQ^2 > PH^2$.

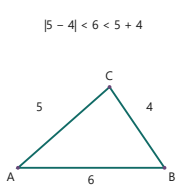
Read the representations

Which Side Lengths Make a Triangle?

For a triangle with three distinct corners, the sum of any two side lengths is strictly greater than the third. Equality makes a flat chain with no interior.

Equivalently, if two sides are a and b and the third is x, then $|a - b| < x < a + b$. The lower bound prevents the shorter side from failing to reach.

In the pictured triangle, sides 5 and 4 permit a third side strictly between 1 and 9. A side of 6 lies in that interval.

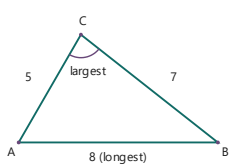


A larger side faces a larger angle

Within one triangle, larger sides lie opposite larger angles. Equal sides face equal angles by the isosceles theorem.

The converse also holds: a larger angle faces a larger side. If that side were equal or smaller, the corresponding angle would be equal or smaller, contradicting the given.

Comparisons are within the same triangle. A longer side in a different triangle does not, by itself, imply a larger opposite angle.



Triangle	Side 1	Side 2	Side 3
ABC	3	4	5
PQR	3	4	5

Can all three matching side lengths establish congruence?

Yes, by SSS. Match the vertex opposite each corresponding side.

THEOREMS AND REPRESENTATIONS

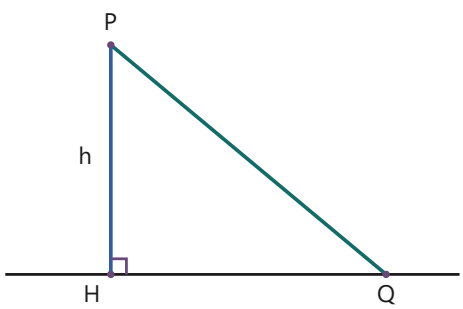
The perpendicular is the shortest route to a line

The perpendicular is the shortest route to a line

Let H be the perpendicular foot from a point P outside a line. Any other point Q on the line gives a right triangle PHQ.

Pythagoras gives $PQ^2 = PH^2 + HQ^2$. If $Q \neq H$, then $HQ^2 > 0$, so $PQ > PH$.

Distance from a point to a line means this perpendicular length PH. The endpoint P must be outside the line for it to be positive.



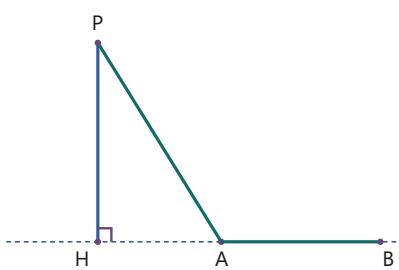
$PQ^2 = PH^2 + HQ^2 > PH^2$ when $Q \neq H$

A segment may stop before the perpendicular foot

A line continues indefinitely. A segment has two endpoints, so the perpendicular foot may lie beyond it.

In this diagram H is outside segment AB. For every point on AB, the horizontal distance from H is at least HA.

Pythagoras then makes PA the shortest distance to the segment. Use the perpendicular foot only when it lies on the target segment.

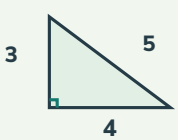


For segment AB, the nearest point is A

Can the shortest walk form a triangle?

Two paths have lengths 7 m and 12 m. A third side x must satisfy $5 < x < 19$. An 18 m path can close a triangle.

At $x = 19$ the paths lie along one straight line. At $x = 5$ the shorter lengths also add exactly to the longest. Neither is a triangle.



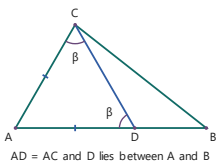
Two triangles each have legs 3 and 4 meeting at 90°. Is that enough?

Yes. Two corresponding sides and their included right angle give SAS.

WORKED EXAMPLES

From the givens to the answer

Justify the side–angle ordering



AD = AC and D lies between A and B

1. Suppose AB > AC in △ABC. Choose D on AB with AD = AC. The smaller triangle ACD is isosceles, so angle ACD = angle ADC.
2. Angle ADC is exterior to △DBC. The triangle angle sum gives angle ADC = angle DBC + angle BCD, which is greater than angle ABC.
3. Angle ACB contains angle ACD as a proper part, so angle ACB > angle ACD = angle ADC > angle ABC. The larger side AB therefore faces the larger angle C.

Is equality enough to make a triangle?

A learner accepts lengths 3, 4 and 7 because 3 + 4 = 7.



3 + 4 = 7: no enclosed region

1. The two shorter sides only reach the endpoints when laid flat on the longest.
2. There is no positive enclosed area and no third vertex away from the base.
3. The triangle inequality is strict: the sum must exceed the third side.

Find the allowed integer third sides

1. Two sides measure 6 cm and 10 cm. The third side x must satisfy $10 - 6 < x < 10 + 6$.
2. Thus $4 < x < 16$. Integer lengths are 5, 6, ..., 15 cm.
3. There are $15 - 5 + 1 = 11$ allowed integers. The endpoints 4 and 16 are excluded.

Compare two paths from the same point

1. P is 12 m perpendicularly from a straight boundary at H. Q is a point 5 m along that boundary from H.
2. The sloping length PQ has square $12^2 + 5^2 = 169$, so PQ = 13 m.
3. The shortest route to the whole boundary is PH = 12 m. The extra horizontal component makes the route to Q longer.

Check another case

Example 1

If AB > BC in △ABC, which angle is larger?

Angle C is greater than angle A · Angle A is greater than angle C · Angle B is necessarily 90° · All angles are equal

Angle C is greater than angle A

AB faces C and BC faces A.

Example 2

Prove the perpendicular is shorter than a sloping segment from P to the same line.

Let H be the perpendicular foot and Q a different point on the line.

Pythagoras gives $PQ^2 = PH^2 + HQ^2$ with $HQ^2 > 0$.

Thus $PQ^2 > PH^2$, so positive lengths give PQ > PH.

Equal angles: 40°, 60°, 80°
↓
Same shape
↓
Size still free

Do three equal angle pairs prove congruence?

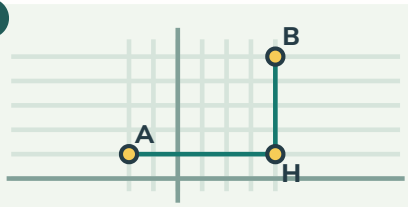
They give similarity. One triangle may be an enlargement of the other.

ILLUSTRATED PROBLEM SHEET

Build the right triangle between two points

The horizontal and vertical displacements are perpendicular. Their squares add to the squared direct distance.

1

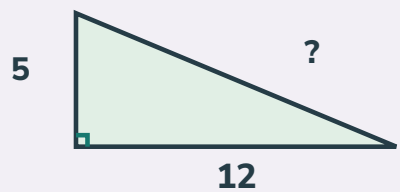


A (-2, 1)
H (4, 1)
B (4, 5)

Find the direct distance from A(−2, 1) to B(4, 5).

The legs are 6 and 4. AB = $\sqrt{52} = 2\sqrt{13}$. The broken route via H is 10 units.

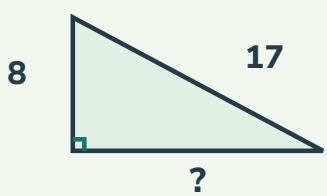
2



A right triangle has legs 5 and 12. How long is its hypotenuse?

$5^2 + 12^2 = 169$, so the longest side is 13.

3



The hypotenuse is 17 and one leg is 8. Recover the other leg.

Subtract squares: 289 − 64 = 225. The missing positive length is 15.

4

$$7^2 + 24^2$$
$$\downarrow$$
$$49 + 576 = 625$$
$$\downarrow$$
$$25^2$$

Can sides 7, 24 and 25 make a right triangle?

Yes. The converse places the right angle opposite the longest side, 25.

Coordinate changes (0,7)
distance 7.

Coordinate changes (−3,4)
distance 5.

CHAPTER PROBLEM

Two sides are 4 cm and 7 cm. How many positive integer third-side lengths are possible?

7. The third side must lie strictly between 3 and 11. Count the integers inside those bounds.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Which lengths form a genuine triangle? A) 2, 5, 7 B) 3, 4, 6 C) 1, 2, 5 D) 4, 4, 9	3, 4, 6 Only 3 + 4 > 6; the other pairs fail the longest-side test.
02 Can sides 8 and 8 enclose a third side of 0.5?	Yes. It is positive and below 16. There is no requirement that the sides be integers.
03 Can lengths 5, 7 and 13 form a triangle? A) Yes, by SSS B) No, because 5 + 7 < 13 C) Yes, because all are positive D) No, because every side must be equal	No, because 5 + 7 < 13 The shorter sides total only 12, less than 13.
04 Two sides are 5 and 9. Which interval describes the third side x? A) $4 < x < 14$ B) $4 \le x \le 14$ C) $0 < x < 4$ D) $x > 14$	$4 < x < 14$ The strict lower and upper bounds are the difference and the sum.
05 Is a route 3 units east and 4 north 7 units from its start?	The route length is 7. The straight-line distance is 5.
06 In △ABC, AB = 9, BC = 6 and CA = 7. Which angle is largest? A) A B) B C) C D) All are equal	C The longest side AB faces angle C.

Third side	3	7	11
Other sides	4 and 7	4 and 7	4 and 7
Triangle?	no	yes	no

Why do endpoints fail?

The third side must be strictly between 3 and 11. Equality collapses the triangle into a straight line.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Within one triangle, angle P is greater than angle Q. Which side comparison follows? A) The side opposite P is longer than the side opposite Q B) The side opposite P is shorter C) All sides are equal D) No side comparison is possible	The side opposite P is longer than the side opposite Q Larger angles face larger sides within the same triangle.
08 A point is 7 cm perpendicularly from a line. What is its shortest distance to that line in cm?	7 The perpendicular itself is the shortest route.
09 Explain why lengths 2, 6 and 8 do not form a genuine triangle.	The two shorter lengths total exactly 8. They can reach across the longest side only in a straight chain. A triangle needs strict inequality and a non-collinear third vertex.
10 Why can the nearest point on a segment be an endpoint rather than a perpendicular foot?	The perpendicular foot lies on the full line, but it may be outside the segment. Only points on the segment are allowed as destinations. When the foot is outside, the nearer endpoint has the smallest displacement along the line and hence the smallest hypotenuse length.
11 Two sides are 8 and 11. What is the largest possible integer third side?	18 $3 < x < 19$, so the largest allowed integer is 18.

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