MATHS FIELD GUIDE · GRADE 9

Triangle congruence and correspondence

OPENING PROBLEM

Can all three matching side lengths establish congruence?

Triangle	Side 1	Side 2	Side 3
ABC	3	4	5
PQR	3	4	5

02 Decide whether the evidence fixes a triangle
03 Rules and worked cases
04 SSS fixes the third vertex up to reflection
05 SSA can leave two possible triangles
06 From the givens to the answer
07 Test an implication in both directions
08 Check your reasoning
09 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Decide whether the evidence fixes a triangle

Correspondence records which vertex matches which. The included angle sits between the named sides.

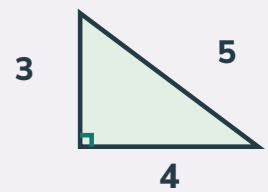
1

Triangle	Side 1	Side 2	Side 3
ABC	3	4	5
PQR	3	4	5

Can all three matching side lengths establish congruence?

Yes, by SSS. Match the vertex opposite each corresponding side.

2



Two triangles each have legs 3 and 4 meeting at 90°. Is that enough?

Yes. Two corresponding sides and their included right angle give SAS.

3

Equal angles: 40°, 60°, 80°

↓

Same shape

↓

Size still free

Do three equal angle pairs prove congruence?

They give similarity. One triangle may be an enlargement of the other.

4

Third side	3	7	11
Other sides	4 and 7	4 and 7	4 and 7
Triangle?	no	yes	no

Why do endpoints fail?

The third side must be strictly between 3 and 11. Equality collapses the triangle into a straight line.

AAA
shape determined, size free.

SSS with fixed lengths
shape and size determined if the triangle exists.

CHAPTER PROBLEM

△PQR ≅ △TSU and SU = 12 cm. Find QR in cm.

12. Q ↔ S and R ↔ U, so QR = SU = 12 cm.

MATHS REFERENCE · GRADE 9

Triangle congruence and correspondence

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
SAS needs an included angle	AB = DE, AC = DF and ∠A = ∠D pair ΔABC with ΔDEF.
RHS	In right triangles, matching hypotenuse 13 and a leg 5 fixes the other leg as 12.
AAA boundary	Triangles with sides 3,4,5 and 6,8,10 have equal matching angles but different sizes.

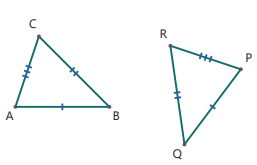
Read the representations

Congruent triangles fit exactly over one another

Congruent triangles have the same shape and size. Sliding, turning or reflecting a triangle preserves its lengths and angles.

In △ABC ≅ △PQR, the order states A ↔ P, B ↔ Q and C ↔ R. Thus AB = PQ, BC = QR and CA = RP, with matching angles equal too.

Side ticks with the same count mark equal lengths. Orientation on the page does not decide the correspondence.



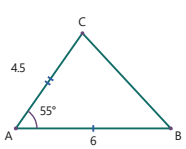
△ABC ≅ △PQR

SAS fixes the opening between two sides

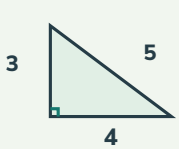
SAS uses two sides and their included angle: the angle where those two sides meet. We accept SAS as the triangle congruence axiom.

Place one known side on its matching side. The included angle fixes the direction of the second side, and its length fixes the third vertex.

An angle elsewhere in the triangle does not give SAS. Once congruence is established, the remaining corresponding parts are equal.



The known angle is between the known sides



Scale every side by two. Which lengths result?

6, 8 and 10. The angles stay fixed, and the perimeter doubles.

THEOREMS AND REPRESENTATIONS

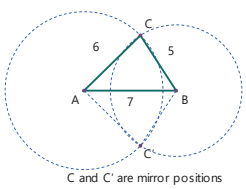
SSS fixes the third vertex up to reflection

SSS fixes the third vertex up to reflection

Place the given base AB. The distances AC and BC put C on a circle about A and a circle about B.

For a valid triangle the circles meet in two positions on opposite sides of AB. Reflecting across AB swaps those positions while preserving every length and angle.

Both positions therefore give congruent triangles. This explains SSS: three matching side lengths determine the shape and size.



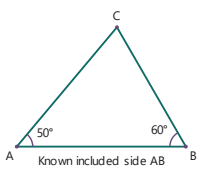
C and C′ are mirror positions

ASA fixes two directions from a known base

With the included side AB fixed, the angle at A fixes one ray and the angle at B fixes the other. Their intersection fixes C. This gives ASA.

With two angles and a non-included side, use the angle sum 180° to find the third angle, then apply ASA. This is AAS.

Two angles alone fix only the shape. A matching side is needed to fix the size.



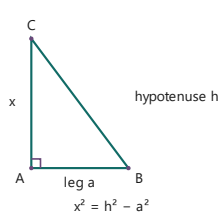
Known included side AB

RHS applies specifically to right triangles

First establish a right angle in both triangles. Then match their hypotenuses and one corresponding leg.

If the hypotenuse is h and the known leg is a, Pythagoras fixes the other leg by $x^2 = h^2 - a^2$. Positive side lengths give the same x in both triangles.

All three sides now match, so SSS proves congruence. The known hypotenuse must be longer than the known leg for a genuine triangle.



$x^2 = h^2 - a^2$

Length	Small	Large
AB ↔ PQ	4	10
BC ↔ QR	6	?

If ABC and PQR are similar in that order, find QR.

The length scale is 10/4 = 2.5, so QR = 15.

THEOREMS AND REPRESENTATIONS

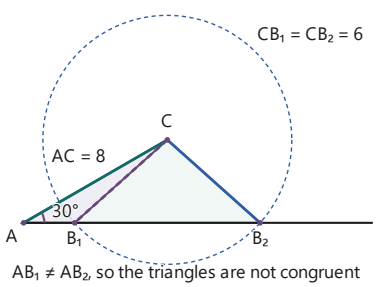
SSA can leave two possible triangles

SSA can leave two possible triangles

Here A is fixed at 30°, AC = 8 and the opposite side CB = 6. B must lie both on the ray from A and on the circle about C of radius 6.

The ray crosses that circle at B_1 and B_2. Both satisfy the given measurements, but AB_1 and AB_2 differ. SSA therefore does not generally guarantee congruence.

Some SSA data do give one triangle. When the opposite known side is longer than AC, A lies inside the circle, so the forward ray has one exit. A known right angle gives the RHS case.



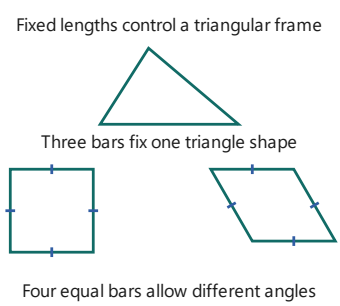
$AB_1 \neq AB_2$, so the triangles are not congruent

Fixed side lengths make a triangle rigid

An ideal frame with three fixed bars joined at their ends has one triangle shape by SSS. It can move or turn, but its angles cannot change without changing a bar length.

A four-bar frame can change its angles while keeping the bar lengths fixed. A diagonal brace divides it into triangles and prevents that motion in the ideal model.

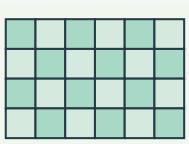
This explains triangular bracing. Actual structures also depend on material strength and the joints.



Use the vertex order to find the side

If triangle ABC is congruent to triangle PQR in that order, A matches P, B matches Q and C matches R. When AB = 5 and AC = 8, PQ = 5 and PR = 8.

Writing QPR in place of PQR changes the matching. Follow the named endpoints before copying a measurement.



This rectangle is a twofold enlargement of a 3 by 2 rectangle. How do the areas compare?

Twenty-four squares versus six. Doubling both directions multiplies area by four.

WORKED EXAMPLES

From the givens to the answer

Do three equal angles prove equal-sized triangles?

A learner calls two equilateral triangles of sides 2 cm and 5 cm congruent because all their angles are 60°.

1. The angle data match, but every side in the second triangle is longer.
2. The triangles have the same shape and different sizes, so they cannot fit exactly.
3. AAA does not prove congruence. A valid congruence condition includes enough length information.

A correspondence gives the missing side

1. Given △DEF ≅ △LNM, write D ↔ L, E ↔ N and F ↔ M.
2. Side EF matches NM. If NM = 9 cm, EF = 9 cm.
3. Angle F matches angle M. If angle M = 47°, angle F = 47°. Keep the ordered match throughout.

Two equal legs with their included right angles

1. Triangles have perpendicular legs 7 cm and 9 cm in each. Match the 7 cm legs, the 9 cm legs and the right angles between them.
2. This is SAS, so the triangles are congruent.
3. RHS would use the hypotenuse and a leg. A right-angle drawing does not turn any two given sides into RHS data.

Check another case

Example 1

A right triangle has hypotenuse 13 cm and one leg 5 cm. Find its other leg in cm.

12

The other leg has square 169 − 25 = 144, hence length 12 cm.

Example 2

State how to finish a proof after establishing △ABC ≅ △DEF, if the required conclusion is AC = DF.

State the congruence condition and the facts that satisfy it.

The ordered correspondence gives A ↔ D and C ↔ F.

Therefore AC = DF as corresponding sides of congruent triangles.

Area ratio = 25:9
↓
Length ratio = 5:3

Why take a square root?

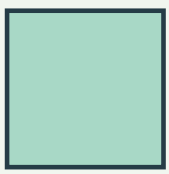
A two-dimensional area uses the length scale twice. Lengths are positive.

ILLUSTRATED PROBLEM SHEET

Test an implication in both directions

A proof uses the stated conditions. A counterexample can defeat a claim that says “always”.

1



square

If a quadrilateral is a square, are its diagonals equal?

Yes. They are hypotenuses of congruent right triangles.

2

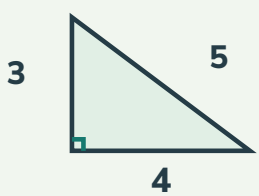


rectangle

If the diagonals of a quadrilateral are equal, must it be a square?

A non-square rectangle is a counterexample. Equal diagonals alone do not force equal sides.

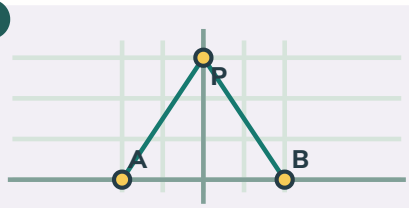
3



Use one diagram to distinguish a theorem from its converse.

A right angle implies $3^2 + 4^2 = 5^2$ for these lengths. The equality also proves the angle is right by the converse of Pythagoras.

4



A (-2, 0)
P (0, 3)
B (2, 0)

Why is P(0, 3) equidistant from A(−2, 0) and B(2, 0)?

The perpendicular bisector is the y-axis. Both squared distances equal 4 + 9 = 13.

A single counterexample
defeats an “always” claim.

A single matching drawing
illustrates the claim but cannot prove it in general.

CHAPTER PROBLEM

Two angles of a triangle are 43° and 72°. Find the third angle in degrees.

65°. The two given angles total 115°. Subtract this from 180° to get 65°.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Given △ABC ≅ △RSP, which side matches BC? A) RS B) SP C) PR D) AB	SP B matches S and C matches P, so BC matches SP.
02 AB = DE, AC = DF and angle BAC = angle EDF. Which condition applies? A) SAS B) SSA C) AAA D) RHS without further information	SAS The equal angle is included between the two specified sides in each triangle.
03 Given △KLM ≅ △XYZ, which angle matches angle L? A) Angle X B) Angle Y C) Angle Z D) Angle K	Angle Y The middle letters L and Y correspond.
04 If ABC is congruent to PQR in that order, which side equals AC?	PR. The first and third vertices correspond in both names.
05 Three corresponding side pairs are equal. Which condition proves congruence? A) SSS B) AAA C) A single angle D) SSA in every case	SSS Three matching sides satisfy SSS.
06 Two angles and the side between them match. Which condition applies? A) RHS B) ASA C) AAA D) Only SSA	ASA The side is included between the two given angles.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Can measuring one drawing prove a relationship for every triangle?	No. State the conditions and derive the relationship for an arbitrary triangle.
08 Which data justify RHS? A) A right angle in each triangle, equal hypotenuses and one equal corresponding leg B) Any equal angle with two sides C) Three equal angles D) Two right angles without lengths	A right angle in each triangle, equal hypotenuses and one equal corresponding leg RHS requires all three named pieces of information.
09 Explain why equal sides of lengths 4, 6 and 8 in two valid triangles establish congruence even if one drawing is reflected.	The three corresponding side lengths agree. SSS establishes congruence. Reflection preserves lengths and angles, so a reversed orientation does not change that result.
10 Why is “two sides and an angle” an incomplete justification for SAS?	The angle must lie between the two given sides. If the angle is elsewhere, the data are SSA and can allow two different triangles. Check the included position before naming SAS.
11 Two triangles have equal angles but all corresponding side lengths are doubled in one. They are... A) Always congruent B) The same shape but different sizes C) Impossible triangles D) Congruent by RHS	The same shape but different sizes Equal angles allow a change of scale. Congruence requires the same size too.

BANANALEARN LEARNING LIBRARY

Where will your maths take you next?

GRADE 9 MATHS

Choose your next problem.

Explore more illustrated guides and practise a related topic.

Browse Grade 9 guides →

Keep exploring

GRADE 9 · TOPIC GUIDE

Isosceles triangles and theorem converses

Open guide →

GRADE 9 · TOPIC GUIDE

Triangle inequalities and shortest distance

Open guide →

Visit bananalearn.com →