MATHS FIELD GUIDE · GRADE 9

Symmetry and tiling with quadrilaterals

OPENING PROBLEM

Why does a parallelogram map to itself under a half-turn?



half-turn

02 Use a diagonal or symmetry to read a quadrilateral
03 Rules and worked cases
04 A tiling covers the plane without gaps or overlaps
05 From the givens to the answer
06 Locate a mirror by equal distances
07 Check your reasoning
08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Use a diagonal or symmetry to read a quadrilateral

Separate statements about side lengths, angles and diagonals.

1

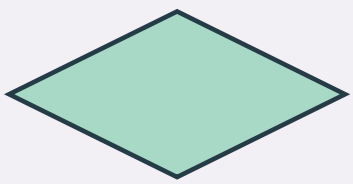


half-turn

Why does a parallelogram map to itself under a half-turn?

Its diagonals bisect each other. Their common midpoint swaps each vertex with its opposite.

2

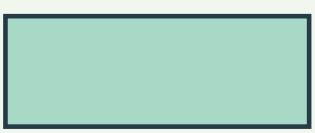


4 equal

Which diagonals split a rhombus into right triangles?

The diagonals meet at right angles and bisect one another.

3

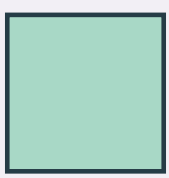


2 mirrors

How many mirror axes does a non-square rectangle have?

Two, through the midpoints of opposite sides. Its diagonals are not mirror axes.

4



4 mirrors

What additional symmetries does a square have?

Both diagonal mirrors also work. A quarter-turn about its centre maps it onto itself.

Non-square rectangle
two mirror axes.

Non-square rhombus
two diagonal mirror axes.

CHAPTER PROBLEM

How many degrees is a half-turn?

180. A half-turn is one-half of a complete 360° turn.

MATHS REFERENCE · GRADE 9

Symmetry and tiling with quadrilaterals

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Rectangle without equal adjacent sides	Two reflection axes pass through midpoints of opposite sides. Its diagonals are not reflection axes.
General parallelogram	A half-turn about the diagonal intersection exchanges opposite vertices and preserves the figure.
Tiling around a point	A quadrilateral's four interior angles sum to 360°. Half-turn copies can place the four corners around a common point.

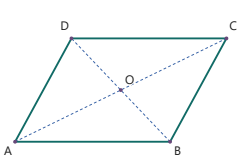
Read the representations

A half-turn swaps opposite vertices of a parallelogram

A half-turn is a rotation of 180° about a centre O. A point and its image lie opposite each other at equal distances from O.

The diagonal intersection of a parallelogram is the midpoint of both diagonals. A half-turn about it swaps A with C and B with D.

Each side lands on its opposite side, so the whole parallelogram fits itself. This is central or half-turn symmetry.



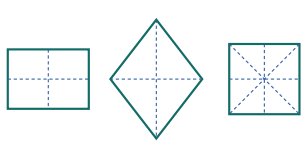
180° about O swaps A with C and B with D

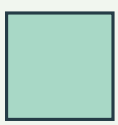
Reflection and rotation are different transformations

A reflection flips points across a line. A rotation turns them about a point. Both preserve lengths and angles.

A non-square rectangle has two reflection axes through opposite side midpoints. A non-square rhombus has two along its diagonals. A square has all four of these axes.

Every parallelogram has half-turn symmetry, but a general oblique parallelogram can have no reflection axis. A square also fits itself after a 90° turn.


Rectangle: 2 Rhombus: 2 Square: 4


4 equal

Is this both a rectangle and a rhombus?

Yes. It has four right angles and four equal sides.

THEOREMS AND REPRESENTATIONS

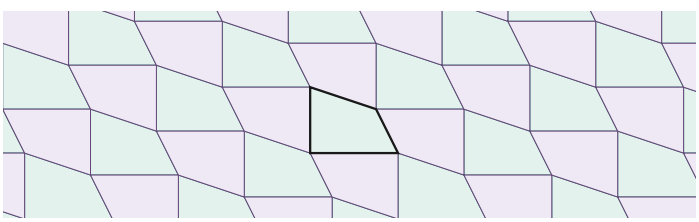
A tiling covers the plane without gaps or overlaps

A tiling covers the plane without gaps or overlaps

A tiling repeats congruent copies to cover the whole plane, with tile interiors neither overlapping nor leaving holes. A finite picture shows only a patch of that infinite pattern.

A tile may be turned or reflected to fit beside a neighbour. It does not need to match its own shape after that turn.

The outlined quadrilateral here is not a parallelogram, yet its copies form a repeating tiling.



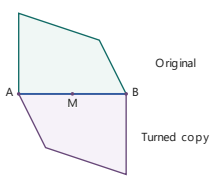
The dark boundary marks one original tile. The pattern continues beyond the picture.

Turning a copy about a side midpoint makes a neighbour

Take a quadrilateral ABCD and the midpoint M of AB. Turn a copy through 180° about M.

A and B swap places, so the copied edge fits AB exactly. Along the shared edge, the two tile interiors lie on opposite sides.

Continue with half-turned copies across boundary sides. Matching edges and a full turn of corner angles allow the pattern to continue.

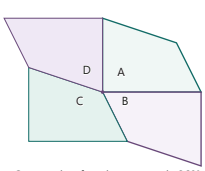


Four corner types fill a complete turn

Label the original quadrilateral's interior angles A, B, C and D. Their sum is 360°.

In this arrangement, the half-turned copies bring one corner of each type to the marked point. They fit around it without an angular gap.

The same edge-matching arrangement repeats along the pattern. The 360° sum explains the corner fit, while matching sides and the repeating construction establish the tiling.



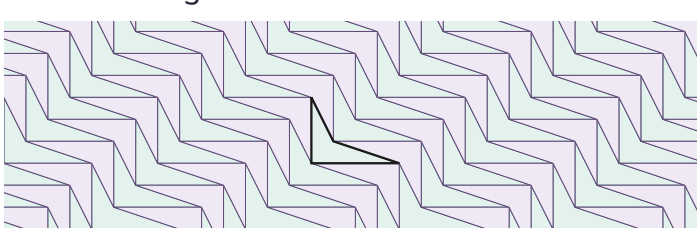
One angle of each type totals 360°

A concave tile also works

A simple concave quadrilateral has one reflex interior angle, but its four interior angles still total 360°.

That reflex corner occupies a larger part of the turn. The other three corner types fill the rest using the same side-midpoint half-turn construction.

The pattern shown uses one concave tile and congruent copies. Crossed or degenerate four-sided drawings are outside this claim.



This patch includes inward corners. Each tile comes from the same original by a half-turn and translations.



right angles

Does a rectangle need four equal sides?

No. It needs four right angles. Opposite sides have matching lengths.

WORKED EXAMPLES

From the givens to the answer

Must a tile have a reflection axis?

A learner says an irregular quadrilateral cannot tile because it has no mirror line.

1. A reflection axis concerns whether the shape maps onto itself.
2. Tiling concerns whether different congruent copies fit next to one another.
3. Half-turns about side midpoints can place neighbouring copies even when a single tile has no reflection symmetry.

Find the image of a point under a half-turn

1. The centre is O(2,−1), and P(5,3) maps to P'. Because O is the midpoint of PP', $(5 + x)/2 = 2$ and $(3 + y)/2 = -1$.
2. Solving gives P'(−1,−5). The coordinate changes from O have reversed signs.
3. Check the averages: $(5 - 1)/2 = 2$ and $(3 - 5)/2 = -1$. A half-turn preserves the distance from O.

Check another case

Example 1

A generic quadrilateral has no symmetry. It can still...

Tile with congruent copies · Have zero interior-angle sum · Map to itself under every rotation · Have crossing sides under the simple-quadrilateral definition

Tile with congruent copies

Copies can be moved into a tiling even without self-symmetry.

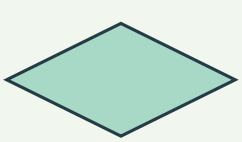
Example 2

Explain why a simple non-degenerate concave quadrilateral can tile the plane with congruent copies despite having a reflex interior angle.

It is a simple non-degenerate quadrilateral with four sides.

Its interior angles, including one reflex angle, still total 360°.

Side-midpoint half-turns and the repeating edge arrangement place congruent copies without requiring convexity.



Must a rhombus have four right angles?

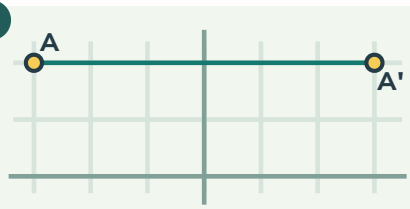
No. Equal sides do not fix a right angle.

ILLUSTRATED PROBLEM SHEET

Locate a mirror by equal distances

A point on the mirror stays fixed. For any other point, the mirror crosses the segment to its image halfway, at a right angle.

1

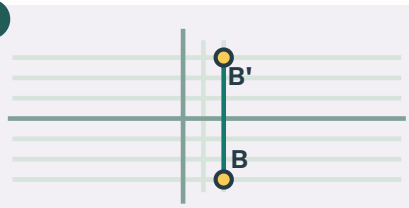


A (-3, 2)
A' (3, 2)

Reflect A(–3,2) in the y-axis.

The height stays two. Three units left becomes three units right, so A' = (3,2).

2

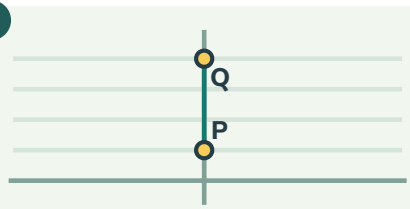


B (2, -3)
B' (2, 3)

Reflect B(2,–3) in the x-axis.

The horizontal position stays two. The sign of the vertical coordinate changes.

3

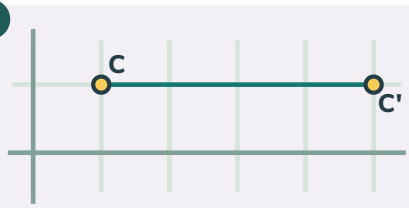


P (0, 1)
Q (0, 4)

What happens to P and Q in the y-axis mirror?

Both are on the mirror line. Each maps to itself.

4



C (1, 1)
C' (5, 1)

Find the vertical mirror taking (1,1) to (5,1).

Halfway from x = 1 to x = 5 is x = 3. Each point is two units from that line.

Mirror in y-axis
(x,y) goes to (–x,y).

Mirror in x-axis
(x,y) goes to (x,–y).

CHAPTER PROBLEM

O is (0,0). A half-turn maps P(4,–7) to P'. Find P''s y-coordinate.

7. A half-turn about the origin reverses both coordinate signs.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Why can copies of any convex quadrilateral tile around a point?	Place one of each of its four angles around that point using half-turned copies along edges. Their sum is 360°.
02 Which statement explains why an irregular quadrilateral can tile? A) Congruent copies can be rotated to match edges and fill full angle turns B) Every quadrilateral has four mirror lines C) Every quadrilateral is a rectangle D) The tile must change its side lengths	Congruent copies can be rotated to match edges and fill full angle turns Tiling allows rigid motions of copies and does not require self-symmetry.
03 Does a mirror image preserve the distances between a shape's vertices?	Yes. Its size and angles stay the same even though orientation reverses.
04 A half-turn centre is O(3,2). If P has x-coordinate 8, find its image's x-coordinate.	-2 $(8 + x)/2 = 3$ gives $x = -2$.
05 A quadrilateral has three interior angles 82°, 91° and 74°. Find its fourth angle in degrees.	113° $360 - 82 - 91 - 74 = 113$.
06 Which is true of every parallelogram? A) It has 180° rotational symmetry B) It has four reflection axes C) It fits itself after every 90° turn D) Its diagonals have equal lengths	It has 180° rotational symmetry The diagonal midpoint is a half-turn centre. The other statements need special conditions.



parallel pairs

Which sides must stay parallel?

Both pairs of opposite sides. Adjacent sides may have different lengths.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 How many reflection axes does a non-square rectangle have? A) 0 B) 1 C) 2 D) 4	2 Its two axes join opposite side midpoints. Diagonals are axes only in the square case.
08 A concave quadrilateral has angles 230°, 45° and 35°. Find its fourth interior angle in degrees.	50° 360 − 230 − 45 − 35 = 50.
09 Why does half-turn symmetry of a parallelogram use its diagonal intersection?	That point bisects both diagonals. A 180° turn sends each endpoint of a diagonal to the other endpoint. All vertices and edges map onto the corresponding opposite vertices and edges of the same shape.
10 Explain why the angle sum 360° alone is not a complete description of a tiling construction.	It explains how the corner angles can fit around a point. The neighbouring side lengths and edges must also match without interior overlap. A repeating placement rule is needed to extend the fit across the plane.
11 P(−2,6) is turned 180° about O(1,1). Find the image's x-coordinate. 	4 x = 2 × 1 − (−2) = 4.

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