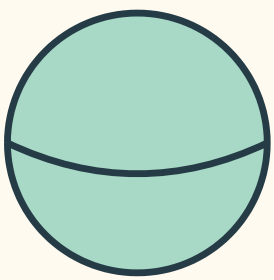
MATHS FIELD GUIDE · GRADE 9

Sphere and hemisphere measures

OPENING PROBLEM

Find the surface and the volume for this sphere.



r = 3

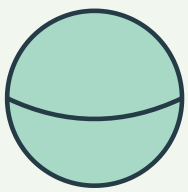
02 Separate the sphere, the dome and the disk
03 Rules and worked cases
04 Sphere surface area matches an enclosing cylinder’s curved area
05 From the givens to the answer
06 Count only the exposed surfaces
07 Check your reasoning
08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Separate the sphere, the dome and the disk

Use $4\pi r^2$ for the whole spherical surface and $(4/3)\pi r^3$ for the whole volume.

1



r = 3

Find the surface and the volume for this sphere.

Surface 36π square units; volume 36π cubic units. The equal numerals describe different quantities.

2

Hemisphere radius 4
↓
Curved surface = $2\pi r^2$
↓
Circular face = πr^2

What is the total exposed area of one free half?

$32\pi + 16\pi = 48\pi$ square units.

3

Diameter doubles
↓
Radius doubles
↓
Volume multiplies by 2^3

How many spheres of radius 1 match the volume of one sphere of radius 2?

Eight, comparing total material without assuming they pack together perfectly.

4

Two hemispheres joined
↓
Two circular faces become internal

What surface remains on the combined sphere?

Only the two curved halves. Each contributes $2\pi r^2$.

Free solid hemisphere
count curved surface and one disk.

Dome painted only on its curved side
omit the disk.

CHAPTER PROBLEM

A sphere has radius 6 cm and volume kπ cm³. Find k.

288. Use $(4/3)\pi r^3$. The coefficient is $(4/3) \times 6^3 = 288$.

MATHS REFERENCE · GRADE 9

Sphere and hemisphere measures

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Sphere r = 3	Surface $4\pi r^2 = 36\pi$; volume $4\pi r^3/3 = 36\pi$. The units distinguish the equal numbers.
Hemisphere curved surface	Half the spherical surface gives $2\pi r^2 = 18\pi$. A flat base adds 9π.
Hemisphere volume	Half the sphere's volume gives 18π cubic units; adding a base face changes no volume.

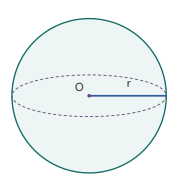
Read the representations

Every sphere surface point is one radius from its centre

A sphere is the set of points in space at distance r from a centre O. The solid ball is the region it encloses.

A plane through O cuts a great circle of radius r. A parallel slice away from O gives a smaller circle.

A sphere has no flat faces or edges. Radius and diameter still satisfy d = 2r, but its surface area and volume are three-dimensional measures.



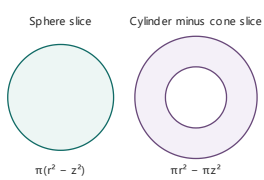
Surface area $4\pi r^2$; volume $4\pi r^3/3$

Compare a sphere slice with a cylinder minus cones

At height z from a sphere's centre, Pythagoras gives the slice radius squared as $r^2 - z^2$. Its disk area is $\pi(r^2 - z^2)$.

Inside a cylinder of radius r and height 2r, put two cones with tips meeting at the centre and bases at the cylinder ends. At that same height z, the removed cone has slice radius $|z|$.

The cylinder-minus-cones slice is an annulus with area $\pi r^2 - \pi z^2$, equal to the sphere's disk area. Equal-area slices at every level give equal volumes.



At matching height z from the centre



r = 3, h = 8

Find the volume of the cylinder.

Base area 9π times height 8 gives 72π cubic units.

THEOREMS AND REPRESENTATIONS

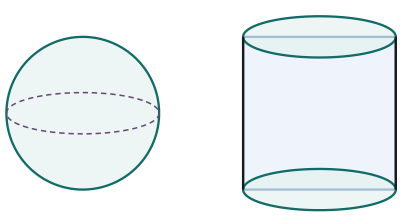
Sphere surface area matches an enclosing cylinder's curved area

Sphere surface area matches an enclosing cylinder's curved area

The geometric surface-area result for a sphere is $S = 4\pi r^2$: four times the area of a radius-r disk.

It equals the curved area of the enclosing radius-r cylinder of height 2r, since $2\pi r \times 2r = 4\pi r^2$. The cylinder's end caps are excluded from this comparison.

This is an equality of areas. A sphere cannot simply be unrolled into a flat rectangle without stretching or cutting and distortion.



Sphere: $4\pi r^2$ Cylinder side: $2\pi r \times 2r$

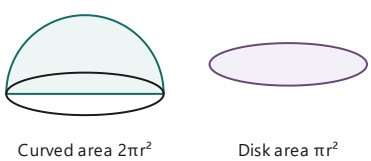
Equal areas; a sphere has no flat net without distortion

A hemisphere adds a flat face only when it is exposed

Cutting a solid ball through its centre makes two equal hemispheres. Each has volume $2\pi r^3/3$ and curved area $2\pi r^2$.

A closed hemisphere also exposes a flat circular disk of area πr^2. Its total area is therefore $3\pi r^2$.

A thin open hemispherical shell has only the curved surface in a one-sided area model. If both sides, thickness or a rim matter, list those surfaces separately.



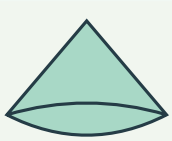
Curved area $2\pi r^2$ Disk area πr^2

Closed total = $3\pi r^2$; open curved shell = $2\pi r^2$

Count the faces exposed by a cut

A sphere of radius 3 cm has surface area 36π cm². Cut it in half and separate the two halves. Each gains a flat face of area 9π cm².

Each hemisphere now exposes $18\pi + 9\pi = 27\pi$ cm². The two together expose 54π cm², an increase of 18π cm².



r = 3, h = 8

How much of the matching cylinder does this cone hold?

One third, or 24π cubic units. The base and perpendicular height must match.

WORKED EXAMPLES

From the givens to the answer

Subtract the double-cone volume to obtain the sphere formula

1. The enclosing cylinder volume is $\pi r^2 \times 2r = 2\pi r^3$. Each removed cone has radius r and height r, so its volume is $\pi r^3/3$.
2. Subtracting both cones gives $2\pi r^3 - 2\pi r^3/3 = 4\pi r^3/3$.
3. This is the sphere volume. The slice comparison matches areas, not the widths or shapes of the two cut sections.

Use radius first and keep the units visible

1. A sphere has diameter 18 cm, so r = 9 cm. Its volume is $(4/3)\pi \times 9^3 = 972\pi$ cm³ and its surface area is $4\pi \times 9^2 = 324\pi$ cm².
2. A hemisphere of the same radius has volume 486π cm³ and curved area 162π cm².
3. If the hemisphere's flat disk is exposed, add 81π cm² to get total area 243π cm². The volume is halved, while the total exposed area is not half the sphere's area.

Is a closed hemisphere's total area half a sphere's area?

A learner halves $4\pi r^2$ and reports $2\pi r^2$ as the total area of a solid hemisphere.

1. That halving gives the curved area only.
2. The cut through the centre exposes a new disk with area πr^2.
3. Adding the disk gives total area $3\pi r^2$. Omit it only if that face is absent, covered, or explicitly excluded.

Check another case

Example 1

A hemisphere has radius 12 cm and volume $k\pi$ cm³. Find k.

1152

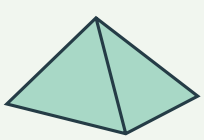
(2/3) × 1728 = 1152.

Example 2

A sphere has radius 8 cm and surface area $k\pi$ cm². Find k.

256

4 × 64 = 256.



B = 18, h = 5

Find the volume without using a slant length.

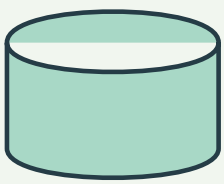
Bh/3 = 30 cubic units. B is the area of the base.

ILLUSTRATED PROBLEM SHEET

Count only the exposed surfaces

List the parts before adding their areas. A joined face disappears inside the finished solid.

1

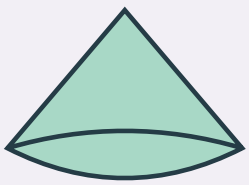


r = 2, h = 5

Which area is exposed on an open-top cylinder with a base?

Curved outside 20π plus base 4π gives 24π square units.

2



r = 3, l = 5

Find the curved area and the total area.

πrl = 15π. Add the circular base 9π for 24π square units.

3



r = 3

A solid hemisphere rests on its flat face. What is its curved area?

Half the full sphere's surface gives $2\pi r^2 = 18\pi$. Count another 9π only if the flat face is exposed.

4

Cylinder r = 2, h = 5

↓

Matching hemisphere on top

↓

20π + 8π + 4π

Find the area including the bottom.

32π square units. Count the cylinder wall, hemispherical dome and bottom. The joining disks are internal.

Remove a cylinder lid
subtract πr^2 from closed surface area.

Join two matching flat disks
subtract $2\pi r^2$ from the sum of exposed areas.

CHAPTER PROBLEM

A closed hemisphere has radius 4 cm and total area kπ cm². Find k.

48. The curved surface contributes $2\pi r^2$ and the base πr^2. Together these give $3 \times 4^2 \times \pi$.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 A hemispherical bowl has internal radius 3 cm. How much can it hold?	$(2/3)\pi \times 27 = 18\pi$ cm³, or 18π mL.
02 If every length is tripled, must the painting cost triple at a fixed price per unit area?	Exposed area scales by nine, so the cost scales by nine.
03 A sphere has radius 5 cm and surface area kπ cm². Find k.	100 $4r^2 = 4 \times 25 = 100$.
04 A sphere has diameter 30 cm and volume kπ cm³. Find k.	4500 Radius 15 gives volume coefficient $(4/3) \times 3375 = 4500$.
05 A hemisphere has radius 3 cm and volume kπ cm³. Find k.	18 Halve the volume of the full sphere. The coefficient is $(2/3) \times 3^3 = 18$.
06 An open thin hemispherical dome has radius 7 cm. Its one-sided curved area is kπ cm². Find k.	98 $2r^2 = 2 \times 49 = 98$.



Find the volume of a hemisphere of this radius.

Half of $(4/3)\pi \times 27$ is 18π cubic units.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 A sphere has surface area 196π cm². Find its radius in cm.	7 $4r^2 = 196$ gives $r^2 = 49$ and positive $r = 7$.
08 Explain how matching cross-sections gives the sphere volume from a cylinder and two cones.	At level z the sphere disk has area $\pi(r^2 - z^2)$. The cylinder with the two central-tip cones removed has annular slice area $\pi r^2 - \pi z^2$ at that level. Equal-area slices give equal volumes, so subtract the two cone volumes from the cylinder to obtain $4\pi r^3/3$.
09 Why does cutting a sphere in half not halve its total exposed surface area?	The curved surface is divided into equal halves. The cut adds a new flat disk to each solid hemisphere. Each closed hemisphere therefore has curved area $2\pi r^2$ plus disk area πr^2.
10 Explain how to choose the correct area formula for a hemispherical object.	Identify whether the requested surface is curved only or also includes the flat circular face. Use $2\pi r^2$ for the curved part and add πr^2 if the disk is exposed and included. If both sides or thickness are relevant, inventory those additional surfaces rather than using a one-sided thin-shell model.
11 A closed hemisphere has radius 10 cm and total area $k\pi$ cm². Find k.	300 The curved surface contributes $2\pi r^2$ and the base πr^2. Together these give $3 \times 10^2 \times \pi$.

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