MATHS FIELD GUIDE · GRADE 9

Solving pairs by substitution

OPENING PROBLEM

Which x satisfies both conditions?

$$y = x + 3$$
$$\downarrow$$
$$x + y = 11$$
$$\downarrow$$
$$x + (x + 3) = 11$$

02 Substitute the whole expression
03 Rules and worked cases
04 From the givens to the answer
05 Decide what the intersection means
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Substitute the whole expression

An expression may replace a variable only when the two are equal. Brackets keep all its terms together.

1

$$y = x + 3$$
$$\downarrow$$
$$x + y = 11$$
$$\downarrow$$
$$x + (x + 3) = 11$$

Which x satisfies both conditions?

2x + 3 = 11 gives x = 4. Then y = 7. Both sum to eleven.

2

$$x = 2y - 1$$
$$\downarrow$$
$$3x + y = 18$$
$$\downarrow$$
$$3(2y - 1) + y = 18$$

Why multiply both terms in the bracket?

The 3 multiplies all of x. This gives 7y – 3 = 18, so y = 3 and x = 5.

3

$$y = 4 - x$$
$$\downarrow$$
$$2x + 2y = 8$$
$$\downarrow$$
$$8 = 8$$

Why does a true constant remain?

The second equation doubles the first relationship. Every point on y = 4 – x works.

4

$$y = 4 - x$$
$$\downarrow$$
$$x + y = 7$$
$$\downarrow$$
$$4 = 7$$

Why is there no solution here?

The first condition fixes the sum at 4. The second asks for a different sum.

y = x, x + y = 8
(4,4).

y = −x, x + y = 8
impossible.

CHAPTER PROBLEM

Solve x = y + 1 and x + y = 9. Find x.

5. 2y + 1 = 9 gives y = 4 and x = 5.

MATHS REFERENCE · GRADE 9

Solving pairs by substitution

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Isolate one variable	For $y = 3x - 1$ and $x + y = 11$, substitute to get $4x - 1 = 11$.
Recover both coordinates	$x = 3$ then $y = 8$. Check $3 + 8 = 11$ and $3 \times 3 - 1 = 8$.
Substitution signals inconsistency	In $y = x + 2$ and $y = x + 5$, equating right sides leaves $2 = 5$.

Read the representations

Replace an unknown by an equal expression

If $x = 2y$, every occurrence of x may be replaced by $2y$ in the other equation.

Substituting into $x + y = 12$ gives $2y + y = 12$.

After finding y, use the original relation to recover x. The solution is a pair.

$$x = 2y$$

$$x + y = 12$$

Replace the highlighted equal quantity

$$2y + y = 12$$

One unknown remains in the second equation.

Choose a useful subject and keep brackets

An equation with coefficient 1 or -1 often gives an easy subject.

From $2x + y = 8$, write $y = 8 - 2x$. If substituting into $3y$, use $3(8 - 2x)$.

Multipliers act on the entire substituted expression, including its constant.

Verification uses both original equations

Valid rearrangement and replacement preserve the simultaneous conditions.

Arithmetic or sign mistakes can still occur, so substitute the proposed pair into both originals.

Do not check only the transformed one-variable equation.

$$x + 7 = 19$$
$$\downarrow$$
$$x = 19 - 7$$
$$\downarrow$$
$$x = 12$$

Which change keeps both sides balanced?

Subtract seven from each side. Substitution gives $12 + 7 = 19$.

WORKED EXAMPLES

From the givens to the answer

Can a multiplier ignore part of a substituted expression?

A learner replaces 2x, with x = y + 3, by 2y + 3.

1. The replacement is 2(y + 3).
2. Distribute to both terms: 2y + 6.
3. The brackets preserve the meaning before expansion.

Solve x = y + 2 and 2x + y = 13

1. Substitute: 2(y + 2) + y = 13.
2. Simplify to 3y + 4 = 13, so y = 3.
3. Recover x = 3 + 2 = 5.
4. Check: 5 = 3 + 2 and 2 × 5 + 3 = 13.

Solve y = 7 − x and 3x − y = 5

1. Substitute: 3x − (7 − x) = 5.
2. This is 4x − 7 = 5, so x = 3.
3. Then y = 4. Check 4 = 7 − 3 and 9 − 4 = 5.

Check another case

Example 1

Which is a correct first substitution for y = 2x − 3 and 5x + 2y = 8?

5x + 4x − 3 = 8 · 5x + 2(2x − 3) = 8 · 5x + 2x − 3 = 8 · 5x + 2(2x) = 8

5x + 2(2x − 3) = 8

The multiplier 2 applies to all of y.

Example 2

Describe the final verification after substitution.

Use the proposed x and y in the first original equation.

Use the same values in the second original equation.

Confirm both equalities, including signs and constants.

3x − 4 = 14
↓
3x = 18
↓
x = 6

Which step undoes multiplication?

Dividing both sides by three. Check: 18 − 4 = 14.

ILLUSTRATED PROBLEM SHEET

Decide what the intersection means

Each equation selects a line. A shared point satisfies both equations.

1

x	0	1	2
y = x + 1	1	2	3
y = 5 − x	5	4	3

Where do the lines meet?

At (2, 3). Both output rows give 3 in the same column.

2

$$y = x + 1$$
$$\downarrow$$
$$x + y = 5$$
$$\downarrow$$
$$x + (x + 1) = 5$$

Substitute to recover the same pair.

2x = 4, so x = 2. Then y = 3.

3

$$x + y = 5$$
$$\downarrow$$
$$-x + y = 1$$
$$\downarrow$$
$$2y = 6$$

Why add the equations?

The x terms cancel. y = 3, then x = 2.

4

Line A	Line B	Solutions
y = 2x + 1	y = 2x + 4	none
y = 2x + 1	2y = 4x + 2	infinite

What changes when the slopes match?

Different intercepts give parallel lines. Matching intercepts give the very same line.

Same slope, different intercepts
no solution.

Different slopes
exactly one solution.

CHAPTER PROBLEM

For y = 2x + 1 and x + y = 16, find x.

5. x + 2x + 1 = 16 gives x = 5.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 A pen costs ₹2 more than a pencil. Two pencils and a pen cost ₹17. Find each price.	Pencil x, pen x + 2. Then 3x + 2 = 17, so the prices are ₹5 and ₹7.
02 If y = 3x − 2, what replaces 4y? A) 12x − 2 B) 12x − 8 C) 7x − 2 D) 3x − 8	12x − 8 4y = 4(3x − 2) = 12x − 8.
03 A notebook and pen cost ₹40. Two notebooks plus that pen cost ₹65. Find each price.	Subtract the totals: notebook ₹25. The pen is ₹15. Check both purchases.
04 For x = 3y and x + y = 20, find y.	5 3y + y = 20, so y = 5.
05 From x − y = 6, which expression equals x? A) y − 6 B) 6 − y C) y + 6 D) −y − 6	y + 6 Add y to both sides.
06 For x = 2y − 1 and x + 2y = 15, find x.	7 4y − 1 = 15 gives y = 4, then x = 7.

What if the variable disappears?

Use y = 2x + 1 in 4x − 2y = −2. The left side simplifies to −2, so every point on y = 2x + 1 satisfies both equations.

Change the right side to 3. Now −2 = 3 is impossible, so the system has no solution.

2(x + 3) = 2x + 6
↓
2x + 6 = 2x + 6
↓
0 = 0

What does this final equality mean?

Every allowed x satisfies the original equation.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 If x = 5 − y, what is 2x − y after substitution? A) 10 − y B) 10 − 3y C) 5 − 3y D) 10 + y	10 − 3y 2(5 − y) − y = 10 − 3y.
08 For y = x − 4 and 2x + y = 14, find y.	2 3x − 4 = 14 gives x = 6, then y = 2.
09 Why is one solved value not the complete answer to a pair of two-variable equations?	The common solution has both an x-coordinate and a y-coordinate. Use the defining relation to recover the other value. Verify the same pair in both original equations.
10 Explain the sign change when substituting y = 6 − x into x − y.	The entire expression replaces y: x − (6 − x). The outside minus changes both signs inside. The result is 2x − 6.
11 For x = y + 4 and x + y = 18, find y.	7 2y + 4 = 18 gives y = 7.

2(x + 3) = 2x + 7
↓
6 = 7

What if the variable disappears into a false statement?

There is no solution. Subtracting the same 2x did not lose one.

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