MATHS FIELD GUIDE · GRADE 9

Solving pairs by elimination

OPENING PROBLEM

What does eliminating both variables tell you?

First	Second	Second − twice first
x + y = 4	2x + 2y = 8	0 = 0
x + y = 4	2x + 2y = 9	0 = 1

02 Make one coefficient cancel
03 Rules and worked cases
04 From the givens to the answer
05 Decide what the intersection means
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Make one coefficient cancel

Multiplying an entire equation by a nonzero number preserves its line.

1

$$2x + y = 11$$
$$\downarrow$$
$$x - y = 1$$
$$\downarrow$$
$$3x = 12$$

Which variable cancels when you add?

The y terms. x = 4 and y = 3. Check the difference is one.

2

$$3x + 2y = 16$$
$$\downarrow$$
$$x + 2y = 8$$
$$\downarrow$$
$$2x = 8$$

Why subtract the second row?

The y coefficients already match. Subtracting gives x = 4, then y = 2.

3

$$x + 2y = 7$$
$$\downarrow$$
$$3x + y = 11$$
$$\downarrow$$
$$3x + 6y = 21$$

How do you prepare matching coefficients?

Triple the first equation. Subtract the second to get 5y = 10, hence y = 2 and x = 3.

4

First	Second	Second − twice first
x + y = 4	2x + 2y = 8	0 = 0
x + y = 4	2x + 2y = 9	0 = 1

What does eliminating both variables tell you?

The first pair describes a single line. The second pair has no shared point.

Equal coefficients with the same sign subtract the equations.

Equal coefficients with opposite signs add the equations.

CHAPTER PROBLEM

Add x + y = 10 and x − y = 4. What is x?

7. 2x = 14 gives x = 7.

MATHS REFERENCE · GRADE 9

Solving pairs by elimination

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Equal coefficients	Subtract x + y = 7 from 3x + y = 13. Then 2x = 6 gives (3,4).
Opposite coefficients	Add 2x − y = 4 and x + y = 5. Obtain x = 3, then y = 2.
Scale first	With 2x + 3y = 12 and x − y = 1, double the second. Subtraction leaves 5y = 10, so (3,2).

Read the representations

Opposite terms cancel when equations are added

In x + y = 8 and x − y = 2, adding the equal left sides and equal right sides gives 2x = 10.

The y and −y terms cancel, so x = 5. Recover y from an original equation.

Keep an original equation together with the sum; the sum alone cannot determine both unknowns.

Add the entire equalities

	x	+ y	=	8
+	x	− y	=	2
	2x	+ 0	=	10

Opposite y-terms sum to zero.

Scaling creates matching coefficients

Multiply every term on both sides of an equation by the same non-zero constant.

Choose multipliers so one variable has equal coefficients for subtraction or opposite coefficients for addition.

The scaled equation has the same solutions because the multiplication is reversible.

Choose addition or subtraction by the signs

Opposite coefficients such as +3y and −3y disappear on addition.

Equal coefficients such as +4x and +4x disappear on subtraction.

When subtracting an equation, subtract every term, including the constant.

x + 7 = 19
↓
x = 19 − 7
↓
x = 12

Which change keeps both sides balanced?

Subtract seven from each side. Substitution gives 12 + 7 = 19.

WORKED EXAMPLES

From the givens to the answer

Can scaling affect only a variable term?

A learner changes x + 2y = 7 to 3x + 6y = 7.

1. Multiplying the equation by 3 must also multiply the right side.
2. The correct equation is 3x + 6y = 21.
3. Leaving 7 unchanged describes a different line.

Solve 2x + y = 11 and x − y = 1

1. Add: 3x = 12, so x = 4.
2. Use x − y = 1 to find y = 3.
3. Check: 8 + 3 = 11 and 4 − 3 = 1.

Solve 2x + 3y = 13 and 3x − 2y = 0

1. Multiply the first equation by 2: 4x + 6y = 26.
2. Multiply the second by 3: 9x − 6y = 0.
3. Add to obtain 13x = 26, so x = 2.
4. Substitute back to get y = 3. Check both originals.

Check another case

Example 1

To eliminate equal non-zero x-coefficients, a useful operation is...

Always add · Subtract one complete equation from the other · Divide both by zero · Remove x only from one side

Subtract one complete equation from the other

Equal coefficients cancel in a difference.

Example 2

State the checks after an elimination calculation.

Recover the other unknown using an original equation.

Substitute the completed pair into each original equation.

Confirm both equalities and any contextual restrictions.

3x − 4 = 14
↓
3x = 18
↓
x = 6

Which step undoes multiplication?

Dividing both sides by three. Check: 18 − 4 = 14.

ILLUSTRATED PROBLEM SHEET

Decide what the intersection means

Each equation selects a line. A shared point satisfies both equations.

1

x	0	1	2
y = x + 1	1	2	3
y = 5 − x	5	4	3

Where do the lines meet?

At (2, 3). Both output rows give 3 in the same column.

2

$$y = x + 1$$
$$\downarrow$$
$$x + y = 5$$
$$\downarrow$$
$$x + (x + 1) = 5$$

Substitute to recover the same pair.

2x = 4, so x = 2. Then y = 3.

3

$$x + y = 5$$
$$\downarrow$$
$$-x + y = 1$$
$$\downarrow$$
$$2y = 6$$

Why add the equations?

The x terms cancel. y = 3, then x = 2.

4

Line A	Line B	Solutions
y = 2x + 1	y = 2x + 4	none
y = 2x + 1	2y = 4x + 2	infinite

What changes when the slopes match?

Different intercepts give parallel lines. Matching intercepts give the very same line.

Same slope, different intercepts
no solution.

Different slopes
exactly one solution.

CHAPTER PROBLEM

Solve x + y = 13 and x − y = 5. Find y.

4. Adding gives x = 9, then y = 4.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Could you multiply just the x term by three to prepare the first equation?	No. Every term on both sides must scale together.
02 Which operation eliminates y from 2x + 3y = 9 and x − 3y = 6? A) Add the equations B) Subtract the second from the first C) Divide one by y D) Delete both y terms without combining constants	Add the equations The opposite 3y coefficients sum to zero.
03 A notebook and pen cost ₹40. Two notebooks plus that pen cost ₹65. Find each price.	Subtract the totals: notebook ₹25. The pen is ₹15. Check both purchases.
04 Solve 3x + y = 14 and x + y = 6. Find x.	4 Subtract to obtain 2x = 8, so x = 4.
05 Which is twice 2x − y = 5? A) 4x − y = 10 B) 4x − 2y = 10 C) 4x + 2y = 10 D) 4x − 2y = 5	4x − 2y = 10 Multiply both variable terms and the constant by 2.
06 Solve 2x + y = 9 and 2x − y = 3. Find x.	3 Addition gives 4x = 12, so x = 3.

$$2(x + 3) = 2x + 6$$
$$\downarrow$$
$$2x + 6 = 2x + 6$$
$$\downarrow$$
$$0 = 0$$

What does this final equality mean?

Every allowed x satisfies the original equation.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 What is (5x + 2y) − (3x − 2y)? A) 2x B) 2x + 4y C) 2x − 4y D) 8x	2x + 4y Subtracting −2y adds 2y to the first 2y.
08 Solve 2x + 3y = 16 and x + y = 6. Find y.	4 Twice the second is 2x + 2y = 12. Subtract to get y = 4.
09 Why must one original relation be retained after adding two equations?	The sum is only one condition on two unknowns. Many pairs may satisfy that sum but not both original equations. Retaining an original equation makes it possible to recover the second value without losing the original restrictions.
10 Explain why multiplying an equation by zero is unsuitable for elimination.	Every term becomes zero. The resulting statement 0 = 0 contains no information about the original line. A non-zero multiplier is needed for a reversible replacement.
11 Solve x + 2y = 11 and x − 2y = 3. Find x.	7 Addition gives 2x = 14, so x = 7.

2(x + 3) = 2x + 7
↓
6 = 7

What if the variable disappears into a false statement?

There is no solution. Subtracting the same 2x did not lose one.

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