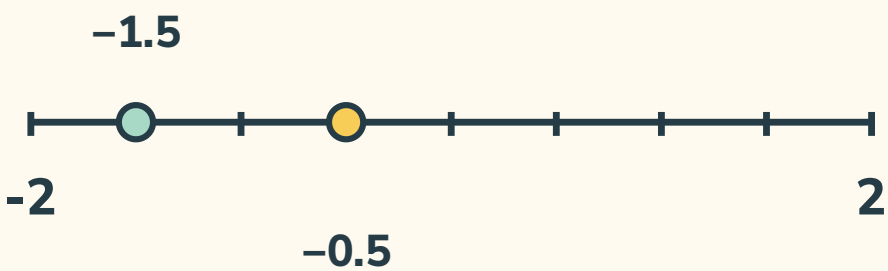
MATHS FIELD GUIDE · GRADE 9

Real numbers on a continuous line

OPENING PROBLEM

Which negative number is greater?



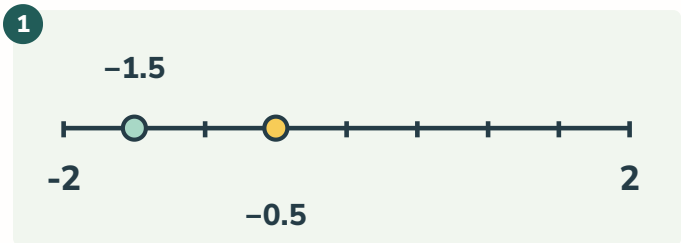
02 Locate, compare and bisect
03 Rules and worked cases
04 From the givens to the answer
05 Bound a root and test the claim
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Locate, compare and bisect

Equal distances on a number line represent equal differences.

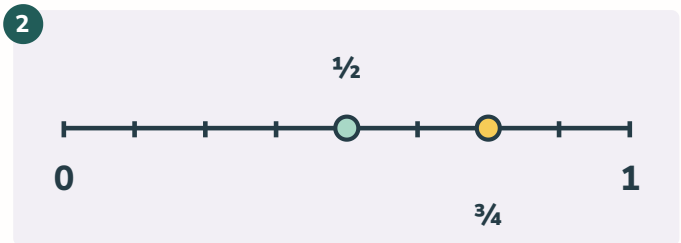
1



Which negative number is greater?

−0.5 lies to the right. Both are negative, but −0.5 is closer to zero.

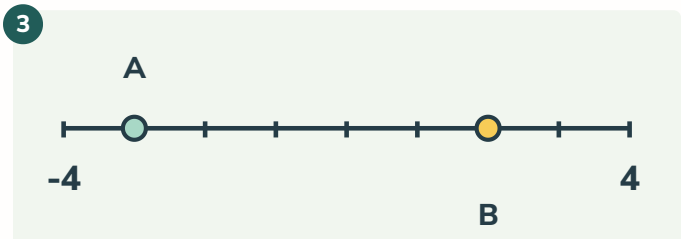
2



Find a rational number between the two marks.

Average 1/2 and 3/4 to obtain 5/8. It lies one eighth from each endpoint.

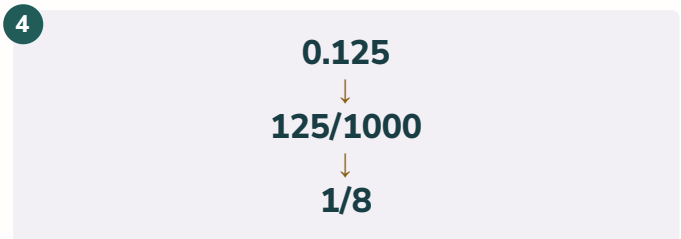
3



How far apart are A and B?

Cross zero: three units to zero and two more to B. Distance is five.

4



Is a terminating decimal a point on the rational line?

Yes. It equals a ratio of integers with nonzero denominator.

Between −1 and 0
−1/2 is one choice.

Between −1 and −1/2
−3/4 is one choice.

CHAPTER PROBLEM

Write 7/20 as a decimal.

0.35. As a fraction in lowest terms the value is 7/20. Dividing numerator by denominator gives 0.35.

MATHS REFERENCE · GRADE 9

Real numbers on a continuous line

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Terminating decimals	0.375 = 375/1000 = 3/8, so its position is rational.
Irrational lengths	$2^2 < 7 < 3^2$ places $\sqrt{7}$ between 2 and 3. No ratio of integers equals it.
Compare negatives	Since $\sqrt{7} > 2.6$ (square both positives), $-\sqrt{7}$ lies left of −2.6.

Read the representations

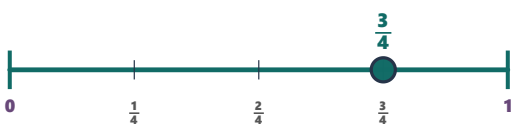
Different names, one position

A number is rational if it can be written p/q, where p and q are integers and q ≠ 0.

For example, 3/4 = 0.75. These are two representations of one number.

Fractions on a line

The unit interval is split into 4 equal steps



3/4 on a number line

Three of four equal intervals from 0 reaches 3/4.

The real-number family

Integers fit inside the rational numbers because n = n/1. Terminating and recurring decimals are rational too.

Irrational numbers have non-terminating, non-recurring decimal expansions. Rational and irrational numbers together form the real numbers.

Real numbers

Rational	Irrational
p/q, integers p and q, q ≠ 0	No fraction of integers
Integers	$\sqrt{2}, \sqrt{3}, \pi$
..., −2, −1, 0, 1, 2, ...	Non-terminating and non-recurring
Also: 2/3, 0.75, 0.333...	

Rational and irrational sets have no shared members. Every point on the real-number line represents a real number.

An exact value and an approximation

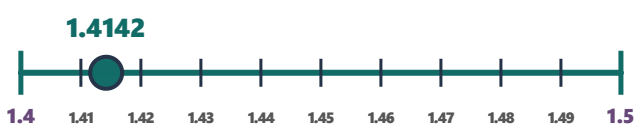
$\sqrt{2}$ is an exact number: its square equals 2. If $\sqrt{2}$ were a fraction in lowest terms, its numerator and denominator would both have to be even.

1.41 is an approximation because $1.41^2 = 1.9881$. Write $\sqrt{2} \approx 1.41$, not $\sqrt{2} = 1.41$.

$$1.4^2 = 1.96 < 2$$
$$1.5^2 = 2.25 > 2$$
$$1.4 < \sqrt{2} < 1.5$$

Decimal number line

Each small step is 0.01



Decimals on a number line

Position shown approximately. The inequalities above use exact squares.

Because $1.4^2 < 2 < 1.5^2$, $\sqrt{2}$ lies between 1.4 and 1.5.

$\sqrt{2} = p/q$
↓
$p^2 = 2q^2$
↓
p is even

Why must p be even?

An odd integer has an odd square. An even square can only come from an even integer.

WORKED EXAMPLES

From the givens to the answer

Does a square-root sign make every number irrational?

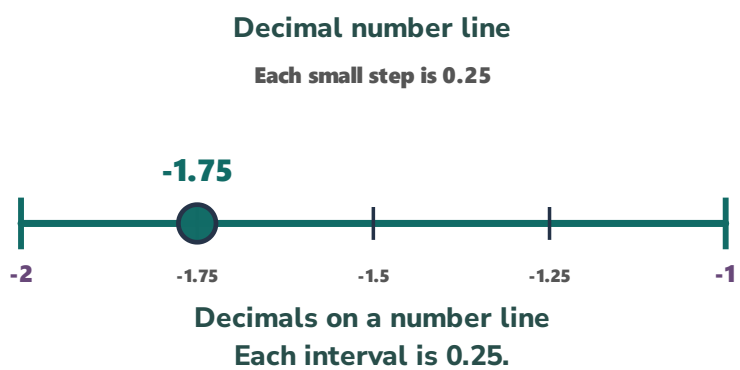
A learner says, "√49 is irrational because it has a root sign."

1. Evaluate the root: $7^2 = 49$, so $\sqrt{49} = 7$.
2. Write 7 as 7/1. It is rational.
3. The value, rather than its notation, determines the number set.

Classify −5, 0.125 and √11

1. −5 = −5/1, so it is an integer and a rational number.
2. 0.125 = 125/1000 = 1/8, so it is rational.
3. 11 is not a perfect square. √11 is irrational.
4. All three belong to the real numbers.

Place −7/4 on a number line



1. −7/4 = −1.75.
2. It lies between −2 and −1, three quarters of a unit to the left of −1.
3. Check that negative values increase to the right.

Check another case

Example 1

Explain the difference between √5 and 2.24.

√5 is exact.

$2.24^2 = 5.0176$, which is not 5.

2.24 is an approximation to √5.

Example 2

Write 0.18 as a fraction.

9/50

18/100 simplifies to 9/50.

$p = 2k$
↓
$4k^2 = 2q^2$
↓
$q^2 = 2k^2$

What does this imply about q?

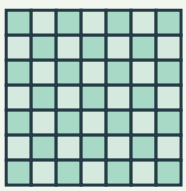
It is even too. Both p and q divisible by 2 contradict lowest terms.

ILLUSTRATED PROBLEM SHEET

Bound a root and test the claim

Squaring a nonnegative number preserves order. The radical sign takes the nonnegative root.

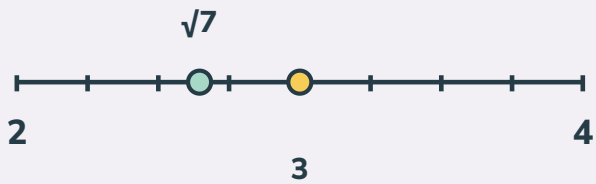
1



Which square has area 49 square units?

Seven rows of seven. Its side length is $\sqrt{49} = 7$.

2



Which integers trap √7?

$2^2 < 7 < 3^2$. Hence $2 < \sqrt{7} < 3$.

3

$$3.3^2 = 10.89$$
$$\downarrow$$
$$11$$
$$\downarrow$$
$$3.4^2 = 11.56$$

Which tenths trap √11?

$3.3 < \sqrt{11} < 3.4$. Compare squares before rounding the root.

4

$$x^2 = 25$$
$$\downarrow$$
$$x = 5 \text{ or } x = -5$$
$$\downarrow$$
$$\sqrt{25} = 5$$

Why are there two equation solutions but one square root symbol value?

Both signs square to 25. The radical denotes the nonnegative one.

$\sqrt{81}$
positive 9.

$x^2 = 81$
two real solutions, 9 and −9.

CHAPTER PROBLEM

Write 0.625 as a fraction.

5/8. As a fraction in lowest terms the value is 5/8. Dividing numerator by denominator gives 0.625.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Which is an integer and therefore rational? A) √7 B) −8 C) π D) √3	−8 −8 = −8/1.
02 Is there a last rational number before 1?	No. Given any rational r below 1, the midpoint (r + 1)/2 is another rational strictly between them.
03 Which value is irrational? A) 0.45 B) −11/3 C) √13 D) √64	√13 13 is not a perfect square, so √13 is irrational. The other values are rational.
04 Which fraction shows that −2.4 is rational? A) −12/5 B) −2/4 C) 12/5 D) −24/100	−12/5 −2.4 = −24/10 = −12/5.
05 Is √(9 + 16) equal to √9 + √16?	No. The first is 5. The second is 3 + 4 = 7.
06 Where does −9/4 lie? A) Between −3 and −2 B) Between −2 and −1 C) Between 2 and 3 D) At −4	Between −3 and −2 −9/4 = −2.25, so it lies between −3 and −2.

r = 3 + √2
↓
√2 = r − 3

Could r be rational?

If so, subtracting the rational number 3 would make √2 rational. That contradicts the proof.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Which statement is correct? A) Every real number is rational B) Every rational number is an integer C) Every integer is rational D) No irrational number is real	Every integer is rational For any integer n, n = n/1.
08 Enter the smaller whole number bounding √30 between consecutive whole numbers.	5 25 < 30 < 36, so 5 < √30 < 6.
09 Explain why a displayed value 3.14159 does not prove that π is rational.	A display shows finitely many digits and may be rounded. The exact π has a non-terminating, non-recurring expansion. The finite decimal 3.14159 itself is rational, but it only approximates π.
10 Explain why 0 belongs to the rational numbers.	0 = 0/1. Both entries are integers. The denominator is non-zero.
11 Which exact value lies between 4 and 5? A) √12 B) √21 C) √25 D) √30	√21 16 < 21 < 25, so 4 < √21 < 5.

$$(\sqrt{2})(\sqrt{2}) = 2$$
$$\downarrow$$
$$\sqrt{2} - \sqrt{2} = 0$$

Can two irrational inputs produce a rational result?

Yes. Both calculations do. The type of each input alone does not decide the output.

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