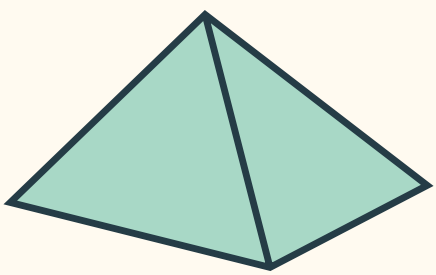
MATHS FIELD GUIDE · GRADE 9

Pyramid volume and surface area

OPENING PROBLEM

The vertical height is 4. Find the volume.



base 6 by 6

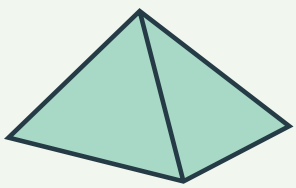
02 Find the height that the formula needs
03 Rules and worked cases
04 Cross-sections extend the rule to other pyramids
05 From the givens to the answer
06 Match the solid to the formula
07 Check your reasoning
08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Find the height that the formula needs

For a pyramid V = Bh/3. A face triangle uses its own perpendicular slant height.

1

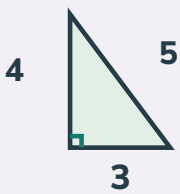


base 6 by 6

The vertical height is 4. Find the volume.

Base area 36. Multiply by 4 and divide by 3 to get 48 cubic units.

2



For that centred square pyramid, what is its face slant height?

From centre to midpoint of a base side is 3. With vertical height 4, the slant height is 5.

3

One face = ½ × 6 × 5
↓
Four faces = 60
↓
Base = 36

Find lateral area and total area.

The side faces total 60 square units. Include the base to obtain 96.

4

Solid	Base	Height
Prism	36	4
Pyramid	36	4

Why are the two volumes different?

The prism keeps full cross-section throughout. Its volume is 144, three times the pyramid's 48.

Square base side 4
base area 16.

Same pyramid with vertical height 9
volume 48.

CHAPTER PROBLEM

A pyramid has base area 45 cm² and vertical height 8 cm. Find its volume in cm³.

120. 45 × 8/3 = 120.

MATHS REFERENCE · GRADE 9

Pyramid volume and surface area

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Volume	Square base side 6 and vertical height 4 give V = 36 × 4/3 = 48.
Slant height of a regular square pyramid	The run from base centre to an edge midpoint is 3. Slant height $\sqrt{(4^2 + 3^2)} = 5$.
Surface area	Four triangles of area 6 × 5/2 plus base 36 give 96 square units.

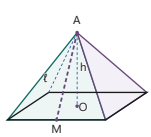
Read the representations

A pyramid joins a polygonal base to an apex

A pyramid has one polygonal base and triangular side faces meeting at one apex. Its vertical height is the perpendicular distance to the base plane.

For a right square pyramid, the apex is above the base centre O. A face's slant height runs to the midpoint M of its base edge.

The slant height, vertical height and sloping corner edge are generally three different lengths.



h reaches base centre O; ℓ reaches edge midpoint M

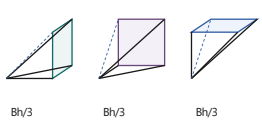
A cube can be partitioned into three congruent pyramids

Choose one cube corner as a common apex and the three faces not containing that corner as bases. Joining the apex to those faces partitions the cube into three pyramids.

Rotating the cube permutes the three pieces, so each has one-third the cube's volume. Each piece has base area a^2 and perpendicular height a.

Thus the one-third rule is exact in this construction: volume = $a^3/3$ = base area × height/3.

Three congruent pieces fill one cube



Each piece has base area a^2 and height a


r = 2, h = 5

Which area is exposed on an open-top cylinder with a base?

Curved outside 20π plus base 4π gives 24π square units.

THEOREMS AND REPRESENTATIONS

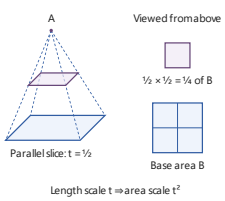
Cross-sections extend the rule to other pyramids

Cross-sections extend the rule to other pyramids

A parallel slice a fraction t of the distance from the apex to the base is a copy of the base scaled in length by t. Its area is t^2 times the base area.

Halfway from apex to base, lengths are halved but slice area is B/4, not B/2. We use the equal-slice principle: solids of equal height with equal cross-sectional areas at every level have equal volume.

Stretch a cube-partition pyramid to the required base area B and height h; its volume remains Bh/3. Any pyramid with the same B and h has the same t^2B slices and hence that volume. Polygonal bases approaching a circle give the cone rule.



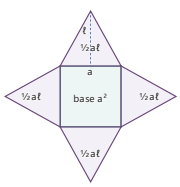
Length scale t ⇒ area scale t^2

A square-pyramid net exposes four triangles

For a right square pyramid of base edge a, the four triangular faces have the same slant height ℓ. Each face has area $a\ell/2$.

Their combined lateral area is $2a\ell$. A closed pyramid adds the square base, giving total area $a^2 + 2a\ell$.

To find ℓ from the vertical height h, use the right triangle with horizontal leg a/2: $\ell^2 = h^2 + (a/2)^2$.



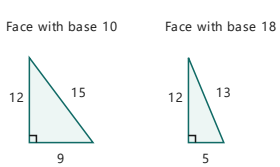
Lateral area $2a\ell$; closed total $a^2 + 2a\ell$

A rectangular base can give two face heights

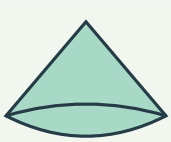
For a right rectangular pyramid with base a by b, faces with base edge a use the horizontal distance b/2 from the centre to that edge.

Those face heights are $\sqrt{[h^2 + (b/2)^2]}$. The faces with base edge b instead use $\sqrt{[h^2 + (a/2)^2]}$.

In the pictured 10 by 18 base with height 12, the two face heights are 15 and 13. Its lateral area is $10 \times 15 + 18 \times 13 = 384$, in square units.



Base 10 × 18; vertical height 12



r = 3, l = 5

Find the curved area and the total area.

πrl = 15π. Add the circular base 9π for 24π square units.

WORKED EXAMPLES

From the givens to the answer

A right square pyramid uses half the base edge

1. A right square pyramid has base edge 10 cm and vertical height 12 cm. Its face slant height is $\sqrt{(12^2 + 5^2)} = 13$ cm.
2. The volume is $(1/3) \times 100 \times 12 = 400$ cm³. The lateral area is $2 \times 10 \times 13 = 260$ cm².
3. Its closed total area is $260 + 100 = 360$ cm². The corner edge would use half the base diagonal instead and is not the face height.

Can the vertical height be used as every triangular face's altitude?

A learner calculates the side-face areas of a square pyramid using h directly.

1. The vertical height usually lies inside the solid and is not in a side-face plane.
2. A triangular face needs its own perpendicular altitude within that face, the slant height to the base-edge midpoint.
3. Recover that slant height from h and half the base edge before adding face areas.

Check another case

Example 1

A pyramid has base area 81 cm² and height 10 cm. Find its volume in cm³.

270

$81 \times 10/3 = 270$.

Example 2

A right square pyramid has base edge 9 cm and face slant height 7 cm. Find its lateral area in cm².

126

$2 \times 9 \times 7 = 126$.



r = 3

A solid hemisphere rests on its flat face. What is its curved area?

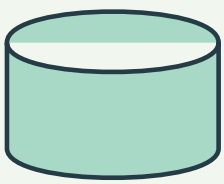
Half the full sphere's surface gives $2\pi r^2 = 18\pi$. Count another 9π only if the flat face is exposed.

ILLUSTRATED PROBLEM SHEET

Match the solid to the formula

Volume uses perpendicular height. A slant length is useful for a curved surface or a triangular face.

1

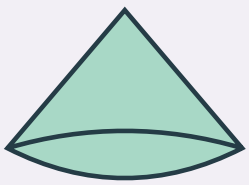


r = 3, h = 8

Find the volume of the cylinder.

Base area 9π times height 8 gives 72π cubic units.

2

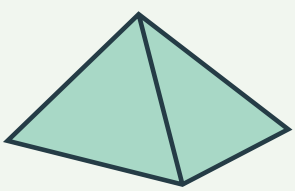


r = 3, h = 8

How much of the matching cylinder does this cone hold?

One third, or 24π cubic units. The base and perpendicular height must match.

3



B = 18, h = 5

Find the volume without using a slant length.

Bh/3 = 30 cubic units. B is the area of the base.

4



r = 3

Find the volume of a hemisphere of this radius.

Half of (4/3)π × 27 is 18π cubic units.

Prism volume V and same base and height pyramid
V/3.

A cylinder of radius r, height 2r
volume $2\pi r^3$.

CHAPTER PROBLEM

A right square pyramid has base edge 6 cm and vertical height 4 cm. Find its face slant height in cm.

5. Use half-edge 3: $\ell = \sqrt{(16 + 9)} = 5$.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 May you use the slant height 5 as h when calculating the pyramid volume?	No. Volume uses the perpendicular separation of apex and base plane.
02 Two cones have equal heights but one has twice the radius. How do volumes compare?	Base area grows fourfold, so volume grows fourfold as well.
03 A pyramid has base area 72 cm² and height 11 cm. Find its volume in cm³.	264 72 × 11/3 = 264.
04 A right square pyramid has base edge 8 cm and vertical height 3 cm. Find its slant height in cm.	5 Half-edge 4 gives √(9 + 16) = 5.
05 A right square pyramid has base edge 12 cm and face slant height 10 cm. Find its lateral area in cm².	240 Four faces give 2aℓ = 2 × 12 × 10 = 240.
06 A closed right square pyramid has base edge 14 cm and face slant height 9 cm. Find its total area in cm².	448 Base 196 plus lateral 2 × 14 × 9 = 252 gives 448.

Cylinder r = 2, h = 5
↓
Matching hemisphere on top
↓
20π + 8π + 4π

Find the area including the bottom.

32π square units. Count the cylinder wall, hemispherical dome and bottom. The joining disks are internal.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 A pyramid has volume 150 cm³ and base area 30 cm². Find its vertical height in cm.	15 150 = 30h/3 = 10h, so h = 15.
08 In a right rectangular pyramid with base a by b, a triangular face with base edge a uses which horizontal leg to recover its slant height? A) The full base diagonal B) b/2 C) a D) b	b/2 The base centre is distance b/2 from an edge of length a.
09 Why do the three cube-partition pyramids each have one-third the cube's volume?	They fill the cube without gaps or interior overlap. Cube rotations permute the three pieces, so they are congruent and have equal volumes. Each has base area a² and height a, giving volume a³/3 = Bh/3.
10 Explain why a right rectangular pyramid can have two different face slant heights.	The apex lies above the base centre. The centre-to-edge distances are b/2 for an edge of length a and a/2 for an edge of length b. Using the same vertical h with these different horizontal legs can produce different hypotenuse lengths.
11 Explain how to find a closed right square pyramid's surface area when only base edge and vertical height are given.	Use half the base edge with the vertical height to find the face slant height by Pythagoras. Add the four triangular face areas, giving 2aℓ. Add the square base area a² if it is exposed.

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