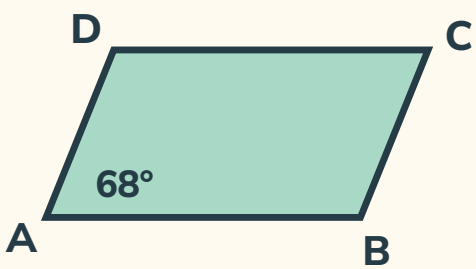
MATHS FIELD GUIDE · GRADE 9

Proofs about lines and angles

OPENING PROBLEM

In parallelogram ABCD, angle A is 68°. Find angle B.



AB ∥ CD

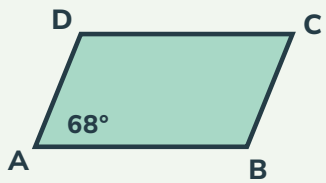
02 Use parallel lines to transfer information
03 Rules and worked cases
04 Prove that vertically opposite angles are equal
05 A perpendicular bisector describes a whole line
06 From the givens to the answer
07 Test an implication in both directions
08 Check your reasoning
09 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Use parallel lines to transfer information

A transversal makes equal corresponding angles with two parallel lines. A parallel cut in a triangle preserves ratios of corresponding lengths.

1



AB ∥ CD

In parallelogram ABCD, angle A is 68°. Find angle B.

Angles on the same side of a transversal between parallel lines sum to 180°. The neighbour is 112°.

2

DE ∥ BC in triangle ABC
↓
AD/AB = AE/AC
↓
3/9 = AE/15

D is on AB and E on AC. Find AE.

AE = 5. The small triangle has one third the full side lengths.

3

Segment	First part	Second part
AB	4	6
AC	6	9

Do these division ratios support a parallel cut?

4/6 = 6/9. For internal D and E on the two sides, the converse gives DE ∥ BC.

4

DE joins side midpoints
↓
Scale = ½
↓
DE = ½ BC

If the opposite side is 18 cm, how long is the midsegment?

9 cm. It is parallel to the opposite side.

Small-to-large scale 1:3
areas in ratio 1:9.

Scale 1:3
perimeters in ratio 1:3.

CHAPTER PROBLEM

One angle in a linear pair is 47°. Find the other angle in degrees.

133°. The angles make a straight angle. The missing part is 180° − 47° = 133°.

MATHS REFERENCE · GRADE 9

Proofs about lines and angles

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Linear pair	Adjacent angles on a straight line total 180°. If one is 37°, its neighbour is 143°.
Vertically opposite angles	Both opposite angles supplement the same adjacent angle, so their measures agree.
Converse for parallel lines	Equal alternate interior angles formed by a transversal prove the two lines are parallel.

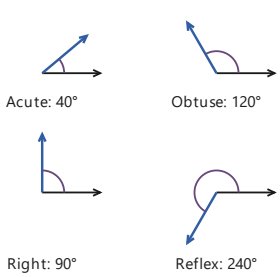
Read the representations

An angle measures a turn between two rays

A ray has one endpoint and extends indefinitely in one direction. Two rays with the same endpoint form an angle.

An acute angle is between 0° and 90°; an obtuse angle is between 90° and 180°. Right, straight and complete angles measure 90°, 180° and 360°.

A reflex angle is between 180° and 360°. The marked arc tells you which turn is intended.

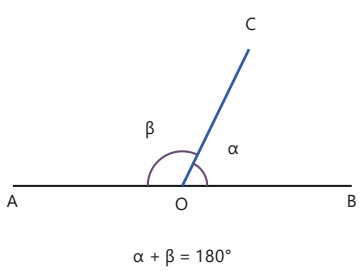


A linear pair fills a straight angle

The angles share a vertex and a side. Their two outer rays point in opposite directions along one line.

Their measures therefore add to 180°: they are supplementary. Conversely, adjacent angles whose measures sum to 180° have opposite outer rays.

Supplementary angles need not be adjacent. Without adjacency, a total of 180° does not make a linear pair.





How much more completes a right angle?

30°. A right angle measures 90°.

THEOREMS AND REPRESENTATIONS

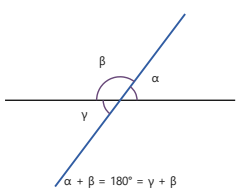
Prove that vertically opposite angles are equal

Prove that vertically opposite angles are equal

Two full lines intersect. The opposite angles α and γ share a supplementary neighbour β.

The linear pairs give α + β = 180° and γ + β = 180°. Subtract β from both equal totals to obtain α = γ.

Repeat the argument for the other opposite pair. Vertically opposite describes position, not whether the angles point upwards.

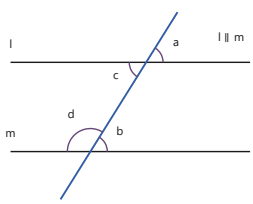


Identify angle positions before choosing a rule

A transversal crosses two lines at distinct points. Corresponding angles occupy the same relative position, like a and b.

Alternate interior angles lie between the lines on opposite sides of the transversal, like b and c. Same-side interior angles lie between the lines on the same side, like c and d.

For parallel lines, corresponding angles are equal. We use this accepted angle rule, equivalent to the Euclidean parallel postulate; its converse is a test for parallelism.

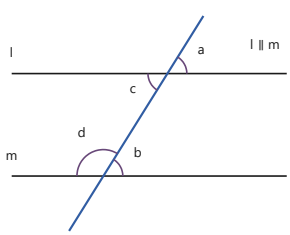


Derive the other parallel-angle rules

In this diagram, a = b by corresponding angles and a = c by vertically opposite angles. Thus b = c: alternate interior angles are equal.

At the lower intersection, b + d = 180° by a linear pair. Since c = b, c + d = 180°: same-side interior angles are supplementary.

Conversely, either equal alternate interior angles or supplementary same-side interior angles leads back to equal corresponding angles. Each condition can therefore establish parallelism.





How much more completes a straight angle?

55°. The adjacent angles together measure 180°.

THEOREMS AND REPRESENTATIONS

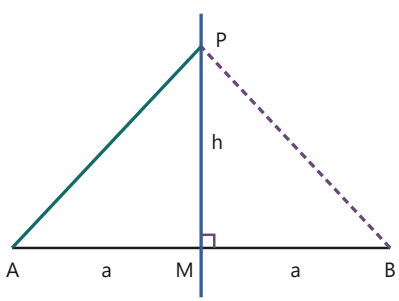
A perpendicular bisector describes a whole line

A perpendicular bisector describes a whole line

A perpendicular bisector passes through the midpoint of a segment at a right angle. Here AM = MB = a and PM = h.

Pythagoras gives $PA^2 = a^2 + h^2 = PB^2$, so PA = PB. This proves equal distance for every point P on the line; for P = M, both distances equal a directly.

Conversely, set A = (−a,0), B = (a,0), a > 0. Equal distances from P(x,y) give $(x + a)^2 + y^2 = (x - a)^2 + y^2$, hence 4ax = 0 and x = 0. P lies on that perpendicular bisector.



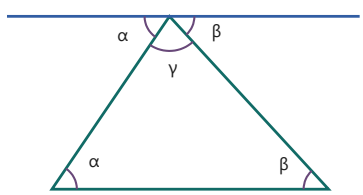
$PA^2 = a^2 + h^2 = PB^2$

A parallel through a vertex explains the triangle angle sum

Draw a line through the top vertex parallel to the base. Alternate angles move the two base-angle measures to that vertex.

The three angles fill a straight angle, so every non-degenerate triangle has angle sum 180°.

A triangle must have at least two acute angles: two angles of 90° or more would leave no positive third angle. Its three angles cannot all be less than 60°, since their sum would be less than 180°.



α + γ + β = 180°

Which points are equally far from A and B?

Place A at (−3, 0) and B at (3, 0). Every point (0, t) has squared distance $9 + t^2$ from each endpoint.

At (1, 0) the distances are 4 and 2. Moving off the perpendicular bisector breaks the equality.



How many such turns make a full turn?

Four. Four times 90° gives 360°.

WORKED EXAMPLES

From the givens to the answer

Does adding to 180° always make a linear pair?

Angles of 70° and 110° are drawn at different vertices. A learner calls them a linear pair.

1. Their measures are supplementary because they sum to 180°.
2. They are not adjacent at one vertex.
3. A linear pair also requires a common side and opposite outer rays.

Use algebra in two adjacent angles

1. Two angles form a linear pair and measure $(3x + 8)°$ and $(2x + 12)°$.
2. $3x + 8 + 2x + 12 = 180$ gives $5x = 160$ and $x = 32$.
3. The angles are 104° and 76°. Check their positive measures and total 180°.

Why parallelism transfers to a third line

1. In one plane, distinct lines l and m are each parallel to n. Suppose l and m meet at P.
2. Neither can meet n, so P is outside n. Both l and m would be parallel lines to n through P.
3. This contradicts the unique-parallel rule. They cannot meet, so $l \parallel m$. Distinctness matters: coincident lines are not called parallel here.

Check another case

Example 1

A line through the midpoint of AB is its perpendicular bisector only if it also...

Makes a right angle with AB · Contains A · Is longer than AB · Passes through the origin

Makes a right angle with AB

Both bisection and perpendicularity are required.

Example 2

Prove that vertically opposite angles α and γ are equal, using a shared neighbour β.

The two linear pairs give $\alpha + \beta = 180°$ and $\gamma + \beta = 180°$.

Their totals are equal.

Subtract β from each side to obtain $\alpha = \gamma$.

Triangle angles	Given	Missing
A	50° and 60°	70°
B	35° and 35°	110°

Why do both rows use 180°?

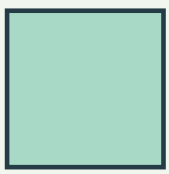
The three interior angles of each plane triangle sum to a straight angle.

ILLUSTRATED PROBLEM SHEET

Test an implication in both directions

A proof uses the stated conditions. A counterexample can defeat a claim that says “always”.

1



square

If a quadrilateral is a square, are its diagonals equal?

Yes. They are hypotenuses of congruent right triangles.

2

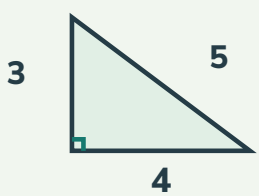


rectangle

If the diagonals of a quadrilateral are equal, must it be a square?

A non-square rectangle is a counterexample. Equal diagonals alone do not force equal sides.

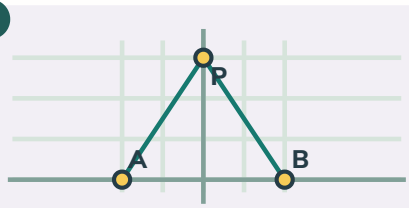
3



Use one diagram to distinguish a theorem from its converse.

A right angle implies $3^2 + 4^2 = 5^2$ for these lengths. The equality also proves the angle is right by the converse of Pythagoras.

4



A (-2, 0)
P (0, 3)
B (2, 0)

Why is P(0, 3) equidistant from A(–2, 0) and B(2, 0)?

The perpendicular bisector is the y-axis. Both squared distances equal 4 + 9 = 13.

A single counterexample
defeats an “always” claim.

A single matching drawing
illustrates the claim but cannot prove it in general.

CHAPTER PROBLEM

Explain why corresponding angles being equal can prove two lines parallel.

Identify that the two angles are in corresponding positions for one transversal. Their equality satisfies the converse of the corresponding-angle rule. Conclude that the two lines are parallel, rather than assuming this before the argument.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 If two corresponding lengths double, must the two triangles be similar?	An additional condition is needed, such as equality of their included angles.
02 Can measuring one drawing prove a relationship for every triangle?	No. State the conditions and derive the relationship for an arbitrary triangle.
03 An angle at two intersecting lines is 126°. Find its vertically opposite angle in degrees.	126° The opposite angle and the given 126° angle each supplement the same adjacent angle. They are equal.
04 Two angles form a linear pair: (4x + 6)° and (2x + 12)°. Find x.	27 6x + 18 = 180 gives x = 27.
05 Parallel lines meet a transversal. One interior angle is 68°. Find its alternate interior partner in degrees.	68° Alternate interior angles are equal when the lines are parallel.
06 Parallel lines meet a transversal. An interior angle is 73°. Find the same-side interior angle in degrees.	107° Same-side interior angles total 180°, so 180 – 73 = 107.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 A 215° angle is... A) Acute B) Right C) Obtuse D) Reflex	Reflex 215° lies between 180° and 360°.
08 A triangle can have... A) Two right angles B) Two obtuse angles C) One obtuse and two acute angles D) Three angles smaller than 60°	One obtuse and two acute angles One angle above 90° can leave two positive acute angles totalling less than 90°. The other cases contradict 180°.
09 A and B are distinct. Explain why every point on their perpendicular bisector is equally far from A and B.	Let M be the midpoint, so AM = BM. For P different from M on the perpendicular, $PA^2 = PM^2 + AM^2$ and $PB^2 = PM^2 + BM^2$. The squared distances, then their non-negative roots, are equal. At P = M the equal halves prove it directly.
10 Two lines look parallel on a small screen. What additional angle evidence would justify the claim?	Choose a transversal crossing both lines at distinct points. Show equal corresponding angles, equal alternate interior angles, or supplementary same-side interior angles. Name the relevant converse to conclude parallelism.
11 A triangle has angles 38° and 61°. Find its third angle in degrees.	81° 180 – 38 – 61 = 81.

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