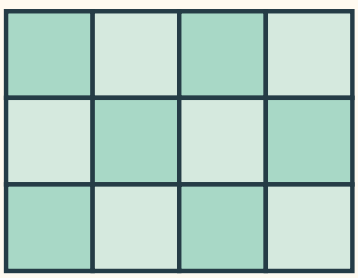
MATHS FIELD GUIDE · GRADE 9

Prisms and cylinders

OPENING PROBLEM

One layer of unit cubes forms this grid. How many cubes?



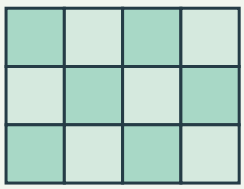
02 Build a solid from equal layers
03 Rules and worked cases
04 From the givens to the answer
05 Match the solid to the formula
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Build a solid from equal layers

Volume = area of one layer × perpendicular height for a prism.

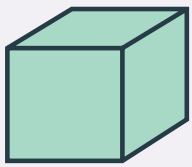
1



One layer of unit cubes forms this grid. How many cubes?

Three rows of four make 12 cubes.

2



How many 1 cm cubes fill this cube?

Nine fit in a layer. Three layers give 27 cm³.

3

Layers	Volume (cm³)
1	12
2	24
5	60

Which box has volume 60 cm³ if its base is 12 cm²?

The height is 5 cm. Divide the volume by the base area.

4

240 cm³
↓
240 mL
↓
0.24 L

How does water help compare capacity and volume?

One cubic centimetre of internal space holds one millilitre.

Base 8 cm², height 3 cm
24 cm³.

Volume 24 cm³, height 6 cm
base 4 cm².

CHAPTER PROBLEM

A prism has cross-section area 18 cm² and perpendicular length 9 cm. Find its volume in cm³.

162. Multiply the constant cross-section by the perpendicular length: 18 × 9 = 162 cubic units.

MATHS REFERENCE · GRADE 9

Prisms and cylinders

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Any right prism	Volume = base area × perpendicular height. A triangular base of area 12 and height 5 gives 60.
Cylinder of r = 3, h = 8	Volume 72π; curved area 2πrh = 48π.
Closed versus open	A closed cylinder adds two disks, an open-top can adds one. For r = 3, each disk adds 9π.

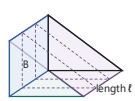
Read the representations

A right prism repeats a constant cross-section

A prism has congruent parallel polygonal end faces. In a right prism, the joining edges are perpendicular to those ends.

A slice parallel to an end face has the same area B throughout the perpendicular length ℓ. Stacking such layers gives volume Bℓ.

For a triangular prism, calculate the triangular end area first. Multiplying three unrelated edge lengths is not generally its volume.



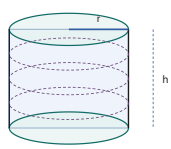
Volume = cross-section area B × length ℓ

A cylinder stacks equal circular disks

A right circular cylinder has radius r and perpendicular height h. Its parallel circular slices all have area πr².

The volume is base area × height = πr²h. A taller cylinder adds more equal layers without changing their disk areas.

For a cylindrical container, capacity uses the internal radius and fill depth. Diameter must be halved before it is squared.



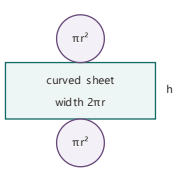
Every layer has area πr² ⇒ volume πr²h

Unrolling the curved wall gives a rectangle

Cut the cylinder wall along one vertical line and unroll it. The rectangle's width is the base circumference 2πr and its height is h.

Its curved area is therefore 2πrh. A closed cylinder adds two disk areas, giving total area 2πrh + 2πr².

An open-top thin cylinder with a base has area 2πrh + πr². A tube open at both ends has only the curved wall in the thin-sheet model.



Curved: 2πrh; closed total: 2πrh + 2πr²


r = 2, h = 5

Which area is exposed on an open-top cylinder with a base?

Curved outside 20π plus base 4π gives 24π square units.

WORKED EXAMPLES

From the givens to the answer

A triangular end changes the base-area calculation

1. A right triangular prism has a right-triangle end with perpendicular legs 6 cm and 8 cm. The end area is $6 \times 8/2 = 24$ cm².
2. The prism's perpendicular length is 11 cm, so its volume is $24 \times 11 = 264$ cm³.
3. For its closed area, add two triangular ends and the rectangles formed by each triangle side. The third end-side length is 10 cm, so the wall area is $(6 + 8 + 10) \times 11$.

Separate the π coefficient from an optional estimate

1. A closed cylinder has radius 3 cm and height 7 cm. Its volume is 63π cm³ and its curved area is 42π cm².
2. Its two caps add 18π cm², so the total area is 60π cm². These expressions are exact for the stated mathematical dimensions.
3. If $\pi \approx 22/7$ is requested, substitute only at the end and label the resulting values as estimates.

Does πd²h use the diameter correctly?

A learner substitutes the diameter directly into $\pi r^2 h$.

1. The formula requires radius, so use $r = d/2$.
2. Squaring the diameter instead multiplies the circular base area by four.
3. For example, a diameter of 10 cm means radius 5 cm, so the base area is 25π cm² rather than 100π cm².

Check another case

Example 1

A cylinder has radius 8 cm and height 3 cm. Its volume is $k\pi$ cm³. Find k.

192

$64 \times 3 = 192$.

Example 2

A closed cylinder has radius 2 cm and height 13 cm. Its total area is $k\pi$ cm². Find k.

60

Curved coefficient 52 plus cap coefficient 8 gives 60.



r = 3, l = 5

Find the curved area and the total area.

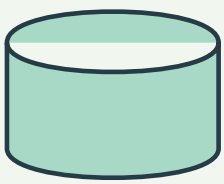
$\pi rl = 15\pi$. Add the circular base 9π for 24π square units.

ILLUSTRATED PROBLEM SHEET

Match the solid to the formula

Volume uses perpendicular height. A slant length is useful for a curved surface or a triangular face.

1

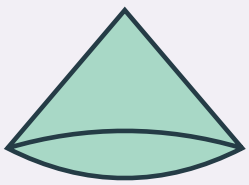


r = 3, h = 8

Find the volume of the cylinder.

Base area 9π times height 8 gives 72π cubic units.

2

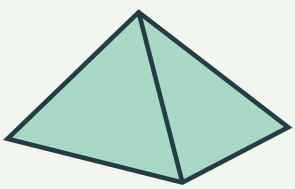


r = 3, h = 8

How much of the matching cylinder does this cone hold?

One third, or 24π cubic units. The base and perpendicular height must match.

3



B = 18, h = 5

Find the volume without using a slant length.

Bh/3 = 30 cubic units. B is the area of the base.

4



r = 3

Find the volume of a hemisphere of this radius.

Half of (4/3)π × 27 is 18π cubic units.

Prism volume V and same base and height pyramid
V/3.

A cylinder of radius r, height 2r
volume $2\pi r^3$.

CHAPTER PROBLEM

A cylinder has radius 4 cm and height 6 cm. Its volume is kπ cm^3. Find k.

96. Use $\pi r^2 h = \pi \times 4^2 \times 6$. The coefficient of π is 96.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Double the three dimensions of a box. How does volume change?	Two times as many cubes fit in each of three directions. The factor is eight.
02 Two cones have equal heights but one has twice the radius. How do volumes compare?	Base area grows fourfold, so volume grows fourfold as well.
03 A right triangular prism has end legs 5 cm and 12 cm and length 8 cm. Find its volume in cm^3.	240 The end area is 30; multiply by length 8 to get 240.
04 A cylinder has diameter 14 cm and height 5 cm. Its volume is $k\pi$ cm^3. Find k.	245 Radius 7 gives $r^2h = 49 \times 5 = 245$.
05 A cylinder has radius 6 cm and height 10 cm. Its curved area is $k\pi$ cm^2. Find k.	120 $2rh = 2 \times 6 \times 10 = 120$.
06 A closed cylinder has radius 5 cm and height 9 cm. Its total area is $k\pi$ cm^2. Find k.	140 Curved coefficient 90 plus two disk coefficients 50 gives 140.



A solid hemisphere rests on its flat face. What is its curved area?

Half the full sphere's surface gives $2\pi r^2 = 18\pi$. Count another 9π only if the flat face is exposed.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 An open-top thin cylindrical pot has radius 4 cm and height 11 cm. Its area, including the base, is kπ cm². Find k.	104 Curved coefficient 88 plus base coefficient 16 gives 104.
08 A cylinder has volume 180π cm³ and radius 3 cm. Find its height in cm.	20 9h = 180, so h = 20.
09 Explain why a right prism's wall area is its end perimeter multiplied by its length.	Each end edge of length s makes a rectangle of area sℓ. All these rectangles have the same perpendicular length ℓ. Their total is ℓ times the sum of the end edges, or perimeter × length.
10 Why is a cylinder's curved wall area based on circumference rather than disk area?	The unrolled wall is a rectangle. Its width is the length around the circular edge, 2πr, and its height is h. Multiplying those two lengths gives 2πrh; πr² belongs to a flat disk.
11 A cylindrical tube is open at both ends. What does the thin-sheet model include, and what changes if its wall has thickness?	The thin-sheet model counts only the curved wall and no circular end disks. With thickness, distinguish the inner and outer curved surfaces. Exposed rims are annuli and need the inner and outer radii if their area is requested.

Cylinder r = 2, h = 5
↓
Matching hemisphere on top
↓
20π + 8π + 4π

Find the area including the bottom.

32π square units. Count the cylinder wall, hemispherical dome and bottom. The joining disks are internal.

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