MATHS FIELD GUIDE · GRADE 9

Prime structure and unique factorisation

OPENING PROBLEM

Read the HCF from the exponents.

Number	Power of 2	Power of 3
24	3	1
36	2	2

02 Use the prime powers as building blocks
03 Rules and worked cases
04 From the givens to the answer
05 Divisibility without listing every multiple
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Use the prime powers as building blocks

Prime exponents record how many copies of each prime a number contains.

1

$$72 = 8 \times 9$$
$$\downarrow$$
$$2^3 \times 3^2$$

Why do two different factor trees end alike?

Their terminal prime factors, counted with multiplicity, are unique apart from order.

2

Number	Power of 2	Power of 3
24	3	1
36	2	2

Read the HCF from the exponents.

Take the smaller exponent in each column. HCF = $2^2 \times 3 = 12$.

3

Number	Power of 2	Power of 3
24	3	1
36	2	2

What is the first shared multiple?

Take the larger exponents. LCM = $2^3 \times 3^2 = 72$.

4

$$108 = 2^2 \times 3^3$$
$$\downarrow$$
$$108 \times 3 = 2^2 \times 3^4$$

Which smallest positive integer makes 108 a square when multiplied by it?

Three supplies the missing partner for the odd exponent. The result is 18^2.

$180 = 2^2 \times 3^2 \times 5$
multiply by 5 for a square.

$180 \div 5 = 36$
a square already.

CHAPTER PROBLEM

Find the HCF of 28 and 42.

14. Both numbers contain the factor 14. Dividing them by it leaves 2 and 3, which have no further common factor.

MATHS REFERENCE · GRADE 9

Prime structure and unique factorisation

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Recognise a prime power	108 = 4 × 27 = $2^2 \times 3^3$. Any full prime factor tree gives these exponents.
HCF versus LCM	For 12 = $2^2 \times 3$ and 18 = 2×3^2, take smaller powers for 6 and larger powers for 36.
Perfect square test	180 = $2^2 \times 3^2 \times 5$. Multiply by 5 to make each exponent even: 900 = 30^2.

Read the representations

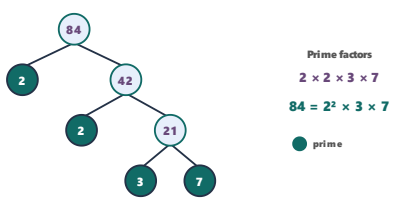
Prime factors are multiplicative building blocks

A prime is an integer greater than 1 with exactly two positive factors: 1 and itself.

Every integer greater than 1 is a product of primes, uniquely apart from their order.

A factor tree can branch differently and still reach the same prime factors.

Factor tree of 84



Prime factorisation

84 = $2^2 \times 3 \times 7$. Every leaf is prime.

Shared factors and enough factors

The HCF uses the minimum exponent of each shared prime. It must divide both numbers.

The LCM uses the maximum exponent of every prime present. Both numbers must divide it.

Prime exponents determine divisibility

Number	Power of 2	Power of 3
18	1	2
24	3	1
HCF: minimum	1	1
LCM: maximum	3	2

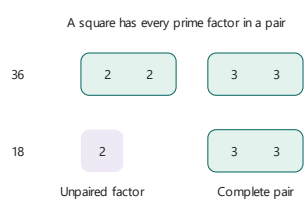
HCF = $2 \times 3 = 6$
LCM = $2^3 \times 3^2 = 72$

For 18 = 2×3^2 and 24 = $2^3 \times 3$, HCF = 6 and LCM = 72.

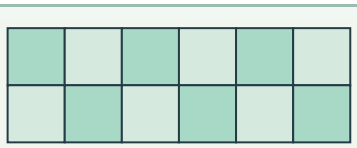
Why squares have paired prime factors

Squaring a prime factorisation doubles every exponent.

Therefore each exponent in a perfect square is even. This fact will support the irrationality proofs.



36 has complete prime-factor pairs. In 18, one factor 2 remains unpaired.



What factor pair builds twelve?

Two rows of six. Turning the rectangle gives the same pair reversed.

WORKED EXAMPLES

From the givens to the answer

Is 1 a prime?

A learner adds 1 to every prime factorisation and calls it a new prime factor.

1. 1 has only one positive factor, so it does not meet the definition of prime.
2. Allowing 1 would make unlimited extra factors of 1 possible.
3. The prime definition keeps the factorisation unique.

Find the HCF and LCM of 40 and 60

1. $40 = 2^3 \times 5$ and $60 = 2^2 \times 3 \times 5$.
2. HCF: choose shared minimum powers, $2^2 \times 5 = 20$.
3. LCM: choose all maximum powers, $2^3 \times 3 \times 5 = 120$.
4. Check 20 divides both and 120 is a multiple of both.

Two lights flash every 12 s and 18 s

1. To find the next shared flash, seek a common multiple.
2. $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$.
3. LCM $= 2^2 \times 3^2 = 36$. If they flash together now, they next coincide in 36 s.

Check another case

Example 1

Find the HCF of 45 and 75.

15

$45 = 3^2 \times 5$ and $75 = 3 \times 5^2$; HCF $= 3 \times 5 = 15$.

Example 2

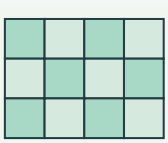
Find the LCM of 9 and 12.

36

$9 = 3^2$ and $12 = 2^2 \times 3$; LCM $= 2^2 \times 3^2 = 36$.

Work backwards and test the limits

Problem	Derivation
Which smallest positive integer must divide 540 to leave a perfect square?	$540 = 2^2 \times 3^3 \times 5$. Remove one 3 and one 5 to leave $2^2 \times 3^2 = 36$. The smallest divisor is 15.
Can 18 divide $2^3 \times 3 \times 5$?	18 needs two factors of 3. The product contains just one, so 18 cannot divide it, even though 2 and 3 both do.
How many positive factors does 72 have?	$72 = 2^3 \times 3^2$. A divisor chooses an exponent 0–3 for 2 and 0–2 for 3. Four choices times three give twelve factors.



Which other pair also uses twelve?

Three and four. The long skinny arrangement 1 by 12 completes the list.

ILLUSTRATED PROBLEM SHEET

Divisibility without listing every multiple

A remainder, a common divisor and a repeated interval answer different questions.

1

$$84 = 2 \times 30 + 24$$
$$\downarrow$$
$$30 = 1 \times 24 + 6$$
$$\downarrow$$
$$24 = 4 \times 6$$

Find HCF(84, 30) by remainders.

The last nonzero remainder is 6. Each division preserves the common divisors.

2

Factor	84	30
2	2 copies	1 copy
3	1 copy	1 copy
Other	7	5

Check the HCF and LCM with prime factors.

HCF = 2 × 3 = 6. LCM = 4 × 3 × 5 × 7 = 420.

3

Two bells: 12 and 18 min
↓
LCM = 36 min

If both ring at 9:00, when next together?

9:36. A shared future time needs a multiple of both intervals.

4

30 red and 42 blue
↓
Identical packs
↓
HCF = 6 packs

How many of each colour in each largest-count pack arrangement?

Five red and seven blue. Every pack has the same contents with none left over.

HCF(18,24)
6.

LCM(18,24)
72; their product is 18 × 24.

CHAPTER PROBLEM

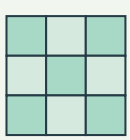
Find the LCM of 8 and 12.

24. The greatest common factor is 4. The least common multiple is (8 × 12) ÷ 4 = 24.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Why is 1 excluded from the primes?	It has only one positive divisor. Allowing extra factors of 1 would also make prime-factor lists non-unique.
02 Can the LCM of two positive integers equal the larger number?	Yes, when the smaller divides the larger. For 6 and 24 it is 24.
03 Which is the prime factorisation of 90? A) 9×10 B) $2 \times 3^2 \times 5$ C) $2^2 \times 3 \times 5$ D) 3×30	$2 \times 3^2 \times 5$ $90 = 2 \times 45 = 2 \times 3 \times 3 \times 5$. Every factor in the final expression is prime.
04 Which number is a perfect square? A) $2^3 \times 3^2$ B) $2^2 \times 3^4$ C) 2×5^2 D) $3^3 \times 7$	$2^2 \times 3^4$ All exponents in $2^2 \times 3^4$ are even.
05 What is the smallest positive integer that makes $75 \times n$ a perfect square?	3 $75 = 3 \times 5^2$. One extra factor 3 makes $3^2 \times 5^2$.
06 There are 32 red and 48 blue beads. Make the greatest number of identical packs with no beads left. How many packs? A) 8 B) 16 C) 32 D) 96	16 The number of packs must divide both counts. HCF(32, 48) = 16.



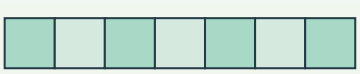
Why does 9 have an odd number of positive factors?

1 and 9 pair together. The middle factor 3 pairs with itself.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Why do different valid factor trees of 60 give the same final primes?	$60 = 2^2 \times 3 \times 5$. Unique prime factorisation fixes the prime factors and their multiplicities. Changing the branching only changes the order and grouping.
08 A learner uses the smallest exponents to find an LCM. Explain the error.	A common multiple must contain enough copies of each prime for both original numbers. Smaller powers can fail divisibility for one number. The LCM therefore uses maximum required exponents.
09 Explain why $2^5 \times 7^2$ is not a perfect square.	A perfect square has even exponents in its prime factorisation. The exponent 5 of prime 2 is odd. At least one factor 2 remains unpaired.
10 Find the HCF of 54 and 72.	18 Both numbers contain the factor 18. Dividing them by it leaves 3 and 4, which have no further common factor.
11 Find the LCM of 15 and 20.	60 The greatest common factor is 5. The least common multiple is $(15 \times 20) \div 5 = 60$.



What makes seven prime?

It has only the pair 1 and 7. No whole rectangle with both sides larger than one uses seven tiles.

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