MATHS FIELD GUIDE · GRADE 9

Parallelograms and their characterisations

OPENING PROBLEM

Why does a parallelogram map to itself under a half-turn?



half-turn

02 Use a diagonal or symmetry to read a quadrilateral
03 Rules and worked cases
04 A diagonal connects two congruent triangles
05 From the givens to the answer
06 Sort shapes by a property that always holds
07 Check your reasoning
08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Use a diagonal or symmetry to read a quadrilateral

Separate statements about side lengths, angles and diagonals.

1

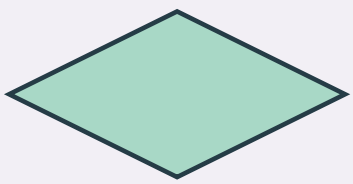


half-turn

Why does a parallelogram map to itself under a half-turn?

Its diagonals bisect each other. Their common midpoint swaps each vertex with its opposite.

2

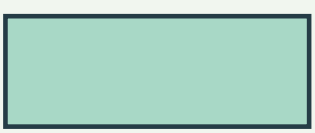


4 equal

Which diagonals split a rhombus into right triangles?

The diagonals meet at right angles and bisect one another.

3

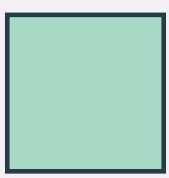


2 mirrors

How many mirror axes does a non-square rectangle have?

Two, through the midpoints of opposite sides. Its diagonals are not mirror axes.

4



4 mirrors

What additional symmetries does a square have?

Both diagonal mirrors also work. A quarter-turn about its centre maps it onto itself.

Non-square rectangle
two mirror axes.

Non-square rhombus
two diagonal mirror axes.

CHAPTER PROBLEM

ABCD is a parallelogram and AB = 14 cm. Find CD in cm.

14. Opposite sides of a parallelogram are equal.

MATHS REFERENCE · GRADE 9

Parallelograms and their characterisations

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
One sufficient test	One opposite pair both parallel and equal is enough to establish a parallelogram.
Diagonals	If diagonals meet at M, AM = MC and BM = MD. Values 2x + 1 and 11 for AM, MC give x = 5.
A tempting false test	Equal diagonals alone also occur in a nonrectangular isosceles trapezium; they do not prove a parallelogram.

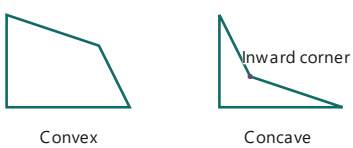
Read the representations

Four straight sides form a quadrilateral

A simple quadrilateral has four distinct vertices joined in boundary order, with no crossing sides and no straight-angle corners.

It can be convex, with every interior angle below 180°, or concave, with one reflex interior angle. A suitable interior diagonal divides either kind into two triangles.

The four interior angles therefore total 360°. A crossed bow-tie is excluded from this definition.

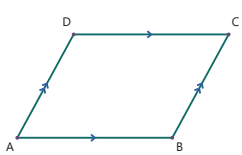


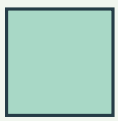
A parallelogram has both opposite side pairs parallel

For vertices A, B, C, D in boundary order, a parallelogram has AB ∥ CD and BC ∥ AD. One parallel pair alone is insufficient.

Arrows identify the parallel pairs. A sloping drawing can still satisfy the definition. Right angles are not required.

Consecutive angles are supplementary by the parallel-line interior-angle rule. Opposite angles are then equal.





square

If a quadrilateral is a square, are its diagonals equal?

Yes. They are hypotenuses of congruent right triangles.

THEOREMS AND REPRESENTATIONS

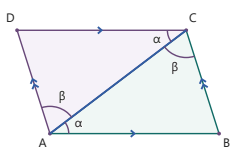
A diagonal connects two congruent triangles

A diagonal connects two congruent triangles

Draw AC. The parallel pairs give angle BAC = angle DCA and angle BCA = angle DAC by alternate interior angles.

AC is shared, so △ABC ≅ △CDA by ASA. The order matches A ↔ C, B ↔ D and C ↔ A.

Corresponding sides give AB = CD and BC = DA. Corresponding angles also give angle B = angle D. These equalities follow from the parallel givens.

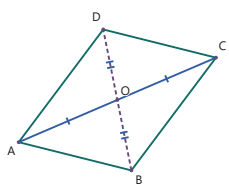


Each diagonal cuts the other into equal halves

Let the diagonals meet at O. Opposite sides AB and CD are equal by the preceding proof.

AB ∥ CD gives equal angles BAO and DCO, and equal angles ABO and CDO. With AB = CD, ASA gives △AOB ≅ △COD.

Thus AO = OC and BO = OD. The two diagonals can have different total lengths while still bisecting each other.



The familiar special cases share the definition

A rectangle adds four right angles to the parallelogram conditions. A rhombus adds four equal sides.

A square satisfies both extra conditions, so it belongs to both families. A non-square rectangle need not have four equal sides.

Use the defining conditions rather than the orientation or the name printed under a picture.

Parallelogram
Both opposite side pairs parallel.

Rectangle
A parallelogram with four right angles.

Rhombus
A parallelogram with four equal sides.

Square
Both a rectangle and a rhombus.



rectangle

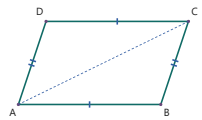
If the diagonals of a quadrilateral are equal, must it be a square?

A non-square rectangle is a counterexample. Equal diagonals alone do not force equal sides.

WORKED EXAMPLES

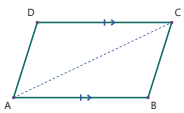
From the givens to the answer

Both opposite side pairs equal are sufficient



1. Suppose a simple quadrilateral has AB = CD and BC = AD. Draw AC, which is shared.
2. SSS gives △ABC ≅ △CDA. Corresponding angles BAC and DCA are equal, so AB ∥ CD by the converse alternate-angle rule.
3. Likewise angles BCA and DAC are equal, giving BC ∥ AD. Both pairs are parallel, so the quadrilateral is a parallelogram.

One opposite pair must be both parallel and equal



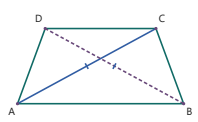
1. Suppose AB ∥ CD and AB = CD. Draw AC. The alternate angles BAC and DCA are equal.
2. AB = CD, shared AC and that included angle give △BAC ≅ △DCA by SAS.
3. The matching angles BCA and DAC are equal, so BC ∥ AD. Together with the given pair, this proves a parallelogram.

Mutual bisection and equal opposite angles give two further tests

1. If AO = OC and BO = OD, vertically opposite angles AOB and COD give SAS congruence. Corresponding angles imply AB ∥ CD. The other triangle pair similarly gives BC ∥ AD.
2. If both opposite angle pairs are equal, write A = C and B = D. The 360° total gives A + B = 180°.
3. The consecutive angle totals then establish both parallel pairs by the converse same-side interior-angle rule. Either full condition is sufficient.

Do equal diagonals alone establish a parallelogram?

A learner identifies a parallelogram only because its two diagonals have the same length.



Equal lengths do not mean mutual bisection

1. The pictured isosceles trapezium has equal diagonals, but its sloping opposite sides are not parallel.
2. Equal diagonal lengths do not say that the diagonals bisect each other.
3. Use a sufficient condition such as mutual bisection or two parallel opposite side pairs.

Check another case

Example 1

Which diagonal condition is sufficient?
Each diagonal bisects the other · Only one diagonal is cut into equal halves · The diagonals look perpendicular · The two diagonals merely intersect

Each diagonal bisects the other

Mutual bisection is the proved characterisation.

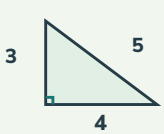
Example 2

A quadrilateral has both opposite angle pairs equal. Explain why it is a parallelogram.

The four angles total 360°, so equality of the opposite pairs makes each consecutive pair total 180°.

These supplementary same-side interior angles establish one pair of parallel sides by a converse angle rule.

Applying the rule to the other consecutive pair gives the second parallel pair.



Use one diagram to distinguish a theorem from its converse.

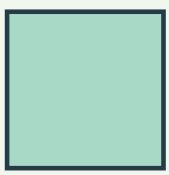
A right angle implies $3^2 + 4^2 = 5^2$ for these lengths. The equality also proves the angle is right by the converse of Pythagoras.

ILLUSTRATED PROBLEM SHEET

Sort shapes by a property that always holds

One shape can belong to more than one family.

1



Is this both a rectangle and a rhombus?

Yes. It has four right angles and four equal sides.

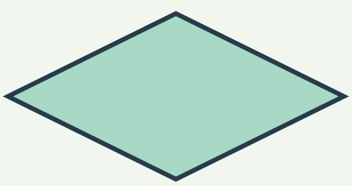
2



Does a rectangle need four equal sides?

No. It needs four right angles. Opposite sides have matching lengths.

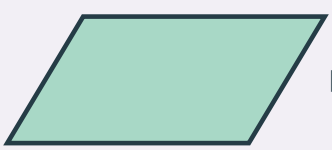
3



Must a rhombus have four right angles?

No. Equal sides do not fix a right angle.

4



Which sides must stay parallel?

Both pairs of opposite sides. Adjacent sides may have different lengths.

Four right angles
a rectangle, possibly a square.

Four equal sides in a quadrilateral
a rhombus, possibly a square.

CHAPTER PROBLEM

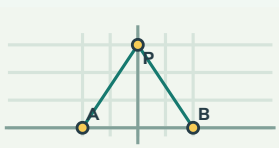
One interior angle of a parallelogram is 68°. Find a neighbouring angle in degrees.

112°. Neighbouring angles total 180°, so 180 − 68 = 112.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Why can copies of any convex quadrilateral tile around a point?	Place one of each of its four angles around that point using half-turned copies along edges. Their sum is 360°.
02 Which condition alone guarantees that a simple quadrilateral is a parallelogram? A) Its diagonals bisect each other B) Its diagonals have equal total lengths C) One opposite pair is parallel D) One angle is acute	Its diagonals bisect each other Mutual bisection gives congruent diagonal triangles and both parallel pairs.
03 A quadrilateral has four equal sides and one right angle. What is its most specific name?	A square. A rhombus is a parallelogram, so the one right angle forces the other three.
04 An interior angle of a parallelogram is 127°. Find its opposite angle in degrees.	127° Opposite angles are equal.
05 In parallelogram PQRS, diagonal PR = 22 cm and diagonals meet at O. Find PO in cm.	11 The diagonals bisect each other, so PO = 22/2 = 11 cm.
06 In parallelogram ABCD, AB = 3x + 2 and CD = 5x – 8. Find x.	5 Opposite sides are equal: 3x + 2 = 5x – 8 gives x = 5.



A (-2, 0)
P (0, 3)
B (2, 0)

Why is P(0, 3) equidistant from A(–2, 0) and B(2, 0)?

The perpendicular bisector is the y-axis. Both squared distances equal 4 + 9 = 13.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Which statement is true for every square? A) It is both a rectangle and a rhombus B) It is a rectangle but cannot be a rhombus C) Its opposite sides are not parallel D) It has exactly one right angle	It is both a rectangle and a rhombus Its four right angles satisfy the rectangle condition and its equal sides satisfy the rhombus condition.
08 Which data satisfy the one-pair parallelogram test? A) AB ∥ CD and AB = CD B) AB ∥ CD and AD = BC only C) AB = CD without a parallel condition D) AC = BD without midpoint information	AB ∥ CD and AB = CD The same pair of opposite sides must be both parallel and equal.
09 Explain why a diagonal of a parallelogram produces two congruent triangles.	Each parallel pair supplies an alternate interior angle equality. The diagonal is the shared included side. ASA proves congruence with the vertex order chosen to match the given angles.
10 Why is checking only one parallel pair insufficient to classify a quadrilateral as a parallelogram?	The definition requires two opposite side pairs to be parallel. A quadrilateral can have just one parallel pair, such as a trapezium with unequal bases. The second parallel pair must be established by a given or a sufficient theorem.
11 In a parallelogram, BO = 9 cm where O is the diagonal intersection. Find BD in cm.	18 O bisects BD, so BD = 2 × 9 = 18 cm.

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