MATHS FIELD GUIDE · GRADE 9

One, none or infinitely many solutions

OPENING PROBLEM

Where do the lines meet?

x	0	1	2
y = x + 1	1	2	3
y = 5 − x	5	4	3

02 Decide what the intersection means
03 Rules and worked cases
04 From the givens to the answer
05 Make one coefficient cancel
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Decide what the intersection means

Each equation selects a line. A shared point satisfies both equations.

1

x	0	1	2
y = x + 1	1	2	3
y = 5 − x	5	4	3

Where do the lines meet?

At (2, 3). Both output rows give 3 in the same column.

2

$$y = x + 1$$
$$\downarrow$$
$$x + y = 5$$
$$\downarrow$$
$$x + (x + 1) = 5$$

Substitute to recover the same pair.

2x = 4, so x = 2. Then y = 3.

3

$$x + y = 5$$
$$\downarrow$$
$$-x + y = 1$$
$$\downarrow$$
$$2y = 6$$

Why add the equations?

The x terms cancel. y = 3, then x = 2.

4

Line A	Line B	Solutions
y = 2x + 1	y = 2x + 4	none
y = 2x + 1	2y = 4x + 2	infinite

What changes when the slopes match?

Different intercepts give parallel lines. Matching intercepts give the very same line.

Same slope, different intercepts
no solution.

Different slopes
exactly one solution.

CHAPTER PROBLEM

For x + y = 3 and x − y = 1, calculate $D = a_1b_2 - a_2b_1$.

-2. $1 \times (-1) - 1 \times 1 = -2$.

MATHS REFERENCE · GRADE 9

One, none or infinitely many solutions

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Unique	$x + y = 5$ and $x - y = 1$ give $2x = 6$, so (3,2).
None	Double $x + y = 5$ to get $2x + 2y = 10$. It contradicts $2x + 2y = 9$.
Infinitely many	$2x + 2y = 10$ is a multiple of $x + y = 5$. One free coordinate determines the other.

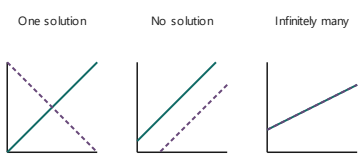
Read the representations

Two proper lines have three possible relationships

Non-parallel lines meet once: one solution. Distinct parallel lines never meet: no solution.

Coincident lines describe the same set of points: infinitely many solutions.

A proper line $ax + by = c$ has at least one non-zero coefficient among a and b.



The three pictures represent one intersection, no intersection, and a shared line. A dashed second colour marks the coincident line.

Compare coefficients using cross-products

For two proper lines $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, let $D = a_1b_2 - a_2b_1$.

If $D \neq 0$, there is one solution. If $D = 0$, the lines are parallel or coincident.

With $D = 0$, they coincide exactly when $a_1c_2 - a_2c_1 = 0$ and $b_1c_2 - b_2c_1 = 0$. Otherwise they are distinct parallel lines.

Two proper lines: division-free tests

Test	Conclusion
$D \neq 0$	One solution
$D = 0$; both constant cross-products are 0	Coincident
$D = 0$; a constant cross-product is non-zero	Distinct parallel

Cross-products avoid dividing by a coefficient that may be zero. These line tests apply only after confirming both equations are proper lines.

An equation with no variables is a separate case

$0x + 0y = 5$ is impossible, so any system containing it has no solution.

$0x + 0y = 0$ is true for every pair and adds no restriction. With one proper line, that leaves infinitely many points on the line.

Two true zero-variable equations permit the whole plane. They are not two coincident lines.

x	−1	0	1
y = 2x + 1	−1	1	3

What stays constant?

Each x-step of one raises y by two. The value at zero is one.

WORKED EXAMPLES

From the givens to the answer

Can zero coefficients be put into a ratio test?

A learner divides by $b_1 = 0$ while comparing two lines.

1. Division by zero is undefined.
2. Use cross-products for proper lines, or rearrange and compare the equations directly.
3. First separate any equation whose x and y coefficients are both zero.

Classify x + 2y = 5 and 2x + 4y = 12

1. Doubling the first gives 2x + 4y = 10, with the same left side as the second.
2. The constants 10 and 12 differ, so no pair can satisfy both.
3. The proper lines are parallel and distinct.

Classify 2y = 6 and x = 4

1. The first gives y = 3, a horizontal line; the second is vertical.
2. They meet at (4,3).
3. Zero coefficients cause no difficulty: $D = 0 \times 0 - 1 \times 2 = -2$, which is non-zero.

Check another case

Example 1

For x + 2y = 6 and 3x + y = 8, calculate D.

-5

$1 \times 1 - 3 \times 2 = -5$.

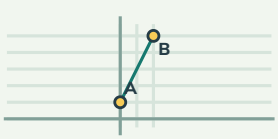
Example 2

Describe a safe order for classifying a pair with zero coefficients.

First check whether either equation has no variable term.

Handle a true or impossible constant statement directly.

For two proper lines, use cross-products or direct comparison without dividing by zero.



A (0, 1)
B (2, 5)

Find rise and run between the points.

Rise 4, run 2. Slope = 4/2 = 2.

ILLUSTRATED PROBLEM SHEET

Make one coefficient cancel

Multiplying an entire equation by a nonzero number preserves its line.

1

$$2x + y = 11$$
$$\downarrow$$
$$x - y = 1$$
$$\downarrow$$
$$3x = 12$$

Which variable cancels when you add?

The y terms. x = 4 and y = 3. Check the difference is one.

2

$$3x + 2y = 16$$
$$\downarrow$$
$$x + 2y = 8$$
$$\downarrow$$
$$2x = 8$$

Why subtract the second row?

The y coefficients already match. Subtracting gives x = 4, then y = 2.

3

$$x + 2y = 7$$
$$\downarrow$$
$$3x + y = 11$$
$$\downarrow$$
$$3x + 6y = 21$$

How do you prepare matching coefficients?

Triple the first equation. Subtract the second to get 5y = 10, hence y = 2 and x = 3.

4

First	Second	Second − twice first
x + y = 4	2x + 2y = 8	0 = 0
x + y = 4	2x + 2y = 9	0 = 1

What does eliminating both variables tell you?

The first pair describes a single line. The second pair has no shared point.

Equal coefficients with the same sign subtract the equations.

Equal coefficients with opposite signs add the equations.

CHAPTER PROBLEM

For 2x + 3y = 5 and 4x + 5y = 9, calculate D.

-2. 2 × 5 − 4 × 3 = −2.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Classify x + y = 4 and 2x + 2y = 8. A) One solution B) No solution C) Infinitely many solutions D) Not real equations	Infinitely many solutions The second is exactly twice the first.
02 A notebook and pen cost ₹40. Two notebooks plus that pen cost ₹65. Find each price.	Subtract the totals: notebook ₹25. The pen is ₹15. Check both purchases.
03 Classify 3x + y = 7 and 6x + 2y = 15. A) Coincident B) Distinct parallel C) Perpendicular by necessity D) One solution	Distinct parallel Twice the first left side equals 14, not 15.
04 Classify y = 2x + 1 and y = 2x − 4. A) One solution B) No solution C) Infinitely many solutions D) The same line	No solution The gradients agree but the intercepts differ.
05 Could you multiply just the x term by three to prepare the first equation?	No. Every term on both sides must scale together.
06 What does 0x + 0y = −2 imply? A) Every pair works B) Exactly one pair works C) No pair works D) It is a vertical line	No pair works It requires the false statement 0 = −2.

y = 3x − 2
↓
10 = 3x − 2
↓
x = 4

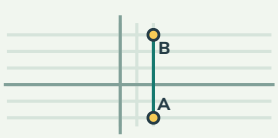
Which input produces output ten?

Undo subtraction, then divide by three. Check: 3 × 4 − 2 = 10.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 A proper line is paired with 0 = 0. How many common real pairs are there? A) None B) One C) Infinitely many D) Exactly two	Infinitely many The true statement removes no point from the proper line.
08 For x = 7 and y = 2, calculate D using coefficients (1,0) and (0,1).	1 1 × 1 − 0 × 0 = 1.
09 Explain why proportional x and y coefficients alone do not prove infinitely many solutions.	The constant must scale by the same factor too. If it does not, the equations impose different totals on the same left side. That produces distinct parallel lines with no common pair.
10 Why are two equations 0 = 0 not described as two coincident lines?	Each statement permits every point in the plane. Neither defines a proper line. Their common set is the whole plane, not a single line.
11 Classify 4x − 2y = 6 and 2x − y = 3. A) One solution B) No solution C) Infinitely many solutions D) The origin only	Infinitely many solutions The first is twice the second, including its constant.



A (2, -2)
B (2, 3)

Does this vertical line have slope zero?

No. Its run is zero, so rise divided by run is undefined. A horizontal line has slope zero.

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