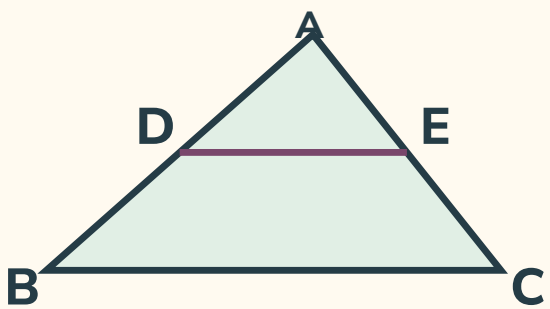
MATHS FIELD GUIDE · GRADE 9

Midpoint theorem, converse and equal intercepts

OPENING PROBLEM

Use AB = 8, AC = 6. AD = 4. Find AE.



DE ∥ BC

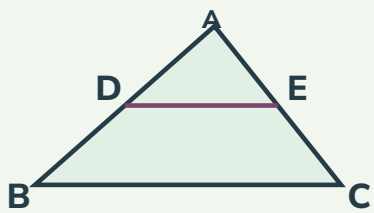
02 A parallel cut creates two similar triangles
03 Rules and worked cases
04 From the givens to the answer
05 Use parallel lines to transfer information
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

A parallel cut creates two similar triangles

D lies on AB, E lies on AC and DE is parallel to BC unless the question asks you to test it.

1



DE ∥ BC

Use AB = 8, AC = 6. AD = 4. Find AE.

D is halfway to B. The smaller triangle is at half scale, so AE = 3.

2

AD = 3, DB = 6
↓
AE = 4
↓
AD/DB = AE/EC

Recover the missing segment EC.

The lower piece is twice the upper piece on AB. EC must be twice 4, which is 8.

3

Side	Upper	Lower
AB	3	5
AC	6	10

If parallelism was not given, can you deduce it?

Yes. 3/5 = 6/10. The converse applies because D and E are internal points.

4

AD/AB = 2/5
↓
Area ADE / Area ABC = 4/25

What fraction of the large area remains below DE?

1 − 4/25 = 21/25. The area factor is the square of the length factor.

AD:DB = 1:3
AD:AB = 1:4.

AD:AB = 3:4
AD:DB = 3:1.

CHAPTER PROBLEM

A triangle's third side is 18 cm. Find the segment joining the other two sides' midpoints in cm.

9. The midsegment has half the third side's length.

MATHS REFERENCE · GRADE 9

Midpoint theorem, converse and equal intercepts

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Forward theorem	Midpoints D and E of AB and AC give DE ∥ BC and DE = BC/2. If BC = 18, DE = 9.
Converse	A line through midpoint D parallel to BC cuts AC at its midpoint E. With AE = 7, AC = 14.
Equal intercepts	Equally spaced parallel lines cut equal successive intervals on any fixed transversal; different transversals can have different interval lengths.

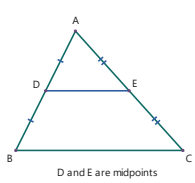
Read the representations

Joining two side midpoints gives a parallel half-length

In △ABC, D is the midpoint of AB and E is the midpoint of AC. The midpoint theorem states DE ∥ BC and DE = BC/2.

Both midpoint givens matter. Being somewhere on each side does not guarantee parallelism or a half-length.

The next construction derives the result using triangle congruence and a parallelogram.



D and E are midpoints

The converse starts with one midpoint and a parallel

Suppose D is the midpoint of AB and the line through D parallel to BC meets AC at E. The converse states that E is the midpoint of AC.

To prove it, choose M as the midpoint of AC. The forward theorem gives DM ∥ BC.

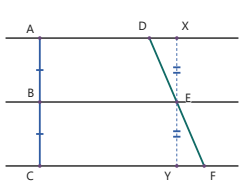
There is only one parallel to BC through D, so DM and DE are the same line. Their intersection with AC is the same point, hence E = M.

Equal intercepts transfer across three parallel lines

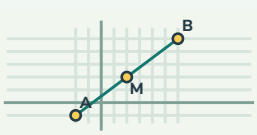
Three parallel lines cut equal successive lengths AB and BC on one transversal. They cut equal successive lengths DE and EF on another transversal.

The conclusion compares lengths on the second transversal: DE = EF. It does not claim that those lengths equal AB.

To justify this, draw XY through E parallel to AC. The parallel boundaries make ABEX and BCYE parallelograms, so EX = AB = BC = EY.



The helper XY is parallel to AC



A (-2, -1)
M (2, 2)
B (6, 5)

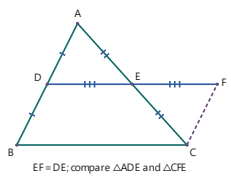
A(−2, −1) and B(6, 5) share which midpoint?

Half the displacement (8, 6) is (4, 3). Add it to A to get M(2, 2).

WORKED EXAMPLES

From the givens to the answer

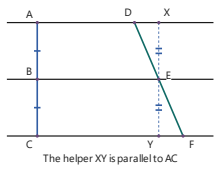
An extension turns the midpoint claim into a proof



EF = DE; compare △ADE and △CFE

1. Extend DE through E to F so EF = DE. Since AE = EC and angles AED and CEF are vertically opposite, △ADE ≅ △CFE by SAS.
2. Thus AD = CF and AD ∥ CF by the matching angles. As AD = DB and A, D, B are collinear, DB is parallel and equal to CF.
3. Therefore DBCF is a parallelogram. Its opposite sides give DF ∥ BC and DF = BC. Since DE = EF, DE = DF/2 = BC/2, with DE ∥ BC.

Finish the equal-intercept argument



The helper XY is parallel to AC

1. In the helper diagram, X, E, Y lie on one line and D, E, F lie on another. Thus angles DEX and FEY are vertically opposite.
2. The outer parallel lines give angle DXE = angle FYE. With EX = EY, ASA gives △DEX ≅ △FEY.
3. Their corresponding sides DE and EF are equal. Equal spacing measured along one transversal transfers to the other.

Does a half-length alone make a midsegment?

A learner measures a segment inside a triangle and finds its length is half the base, then calls both its endpoints midpoints.

1. A length equality alone does not locate endpoints on the two side midpoints.
2. The forward theorem starts with both midpoint positions. The converse starts with one midpoint and a parallel line.
3. Use those conditions before claiming midpoint locations.

Use the theorem with a fresh algebraic length

1. A triangle's midsegment is 2x + 1 cm and its parallel third side is 5x − 4 cm.
2. The third side is twice the midsegment, so 5x − 4 = 2(2x + 1). This gives x = 6.
3. The lengths are 13 cm and 26 cm. Both are positive and satisfy the required factor of two.

Check another case

Example 1

In △ABC, D lies on AB and E lies on AC. Which data justify that E is a midpoint by the midpoint converse?

D is the midpoint of AB and DE ∥ BC · DE is half as long as some unrelated side · D is merely on a side · The picture looks balanced

D is the midpoint of AB and DE ∥ BC

In △ABC, D is the midpoint of AB, E lies on AC and DE ∥ BC. The midpoint converse makes E the midpoint of AC.

Example 2

In △ABC, D and E are the midpoints of AB and AC. Extend DE beyond E to F with EF = DE. Explain how this equality is used to prove DE = BC/2.

It makes E the midpoint of the extended segment DF.

Together with AE = EC and the vertical angle, it gives SAS congruence.

After proving DF = BC, it gives DE = DF/2 = BC/2.



A (0, 0)
P (3, 2)
B (9, 6)

For A(0, 0) and B(9, 6), where is the 1:2 division point from A?

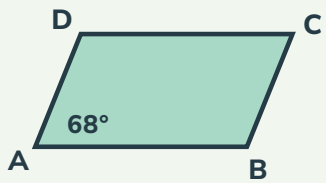
Travel one of three equal steps, so P = (3, 2).

ILLUSTRATED PROBLEM SHEET

Use parallel lines to transfer information

A transversal makes equal corresponding angles with two parallel lines. A parallel cut in a triangle preserves ratios of corresponding lengths.

1



AB ∥ CD

In parallelogram ABCD, angle A is 68°. Find angle B.

Angles on the same side of a transversal between parallel lines sum to 180°. The neighbour is 112°.

2

DE ∥ BC in triangle ABC
↓
AD/AB = AE/AC
↓
3/9 = AE/15

D is on AB and E on AC. Find AE.

AE = 5. The small triangle has one third the full side lengths.

3

Segment	First part	Second part
AB	4	6
AC	6	9

Do these division ratios support a parallel cut?

4/6 = 6/9. For internal D and E on the two sides, the converse gives DE ∥ BC.

4

DE joins side midpoints
↓
Scale = ½
↓
DE = ½ BC

If the opposite side is 18 cm, how long is the midsegment?

9 cm. It is parallel to the opposite side.

Small-to-large scale 1:3
areas in ratio 1:9.

Scale 1:3
perimeters in ratio 1:3.

CHAPTER PROBLEM

A triangle's midsegment is 7 cm. Find its parallel third side in cm.

14. The third side is twice the midsegment: 2 × 7 = 14 cm.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 May you set AD/DB equal to AE/AC for a parallel cut?	Those fractions compare different parts. Match part to remainder on both sides, or part to whole on both.
02 D is the midpoint of AB. A line through D parallel to BC meets AC at E. What follows? A) E is the midpoint of AC B) E must be the vertex C C) DE must equal BC D) AB and AC must be equal	E is the midpoint of AC This is the converse of the midpoint theorem.
03 If two corresponding lengths double, must the two triangles be similar?	An additional condition is needed, such as equality of their included angles.
04 The side parallel to a triangle's midsegment is 31 cm. Find the midsegment length in cm.	31/2 The midpoint theorem gives 31/2 cm.
05 D is the midpoint of AB and DE ∥ BC, with E on AC. If AE = 8 cm, find AC in cm.	16 The converse makes E the midpoint, so AC = 2 × 8 = 16 cm.
06 A midsegment is x + 3 cm and the parallel side is 3x + 1 cm. Find x.	5 3x + 1 = 2(x + 3) gives x = 5.

A = (−3, 1)
↓
M = (1, 4)
↓
B = (5, 7)

Recover the far endpoint from a midpoint.

From A to M add (4, 3). Add the same vector again to reach B.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Which condition is required by the forward midpoint theorem? A) Both endpoints are side midpoints B) Only the segment length is known C) The triangle is isosceles D) The base is horizontal on the page	Both endpoints are side midpoints The theorem applies to the segment joining two side midpoints in any triangle.
08 Three parallel lines cut equal intercepts on one transversal. On another, the successive intercepts are $3x - 2$ and $x + 8$. Find x.	5 The intercepts on the second transversal are equal, so $3x - 2 = x + 8$ and $x = 5$.
09 Explain why equal intercepts on two different transversals need not have the same numerical length.	The theorem equates the two successive intercepts within each transversal. A sloping transversal can travel farther between the same parallel boundaries. It follows that DE = EF, but AB = DE needs extra information.
10 Prove the midpoint converse by introducing the actual midpoint M of the second side.	Let M be the midpoint of AC and use the forward theorem to get DM ∥ BC. The given DE is also parallel to BC through D. Uniqueness of the parallel through D makes E and M the same intersection with AC, so E is its midpoint.
11 A triangle has a 23 cm midsegment. Find the parallel third side in cm.	46 The third side is $2 \times 23 = 46$ cm.



A (0, 0)
M (2, 2)
C (4, 4)

How can midpoints test a quadrilateral?

Compare both diagonal midpoints. If both bisect each other in a simple quadrilateral, it is a parallelogram.

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