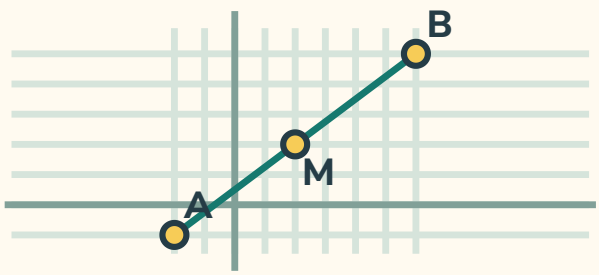
MATHS FIELD GUIDE · GRADE 9

Midpoint and coordinate shape checks

OPENING PROBLEM

A(−2, −1) and B(6, 5) share which midpoint?



A (-2, -1)
M (2, 2)
B (6, 5)

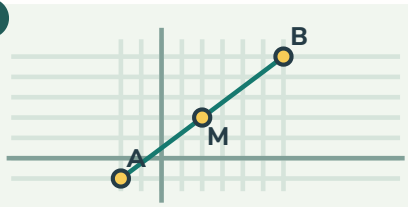
02 Locate a point along a segment
03 Rules and worked cases
04 Collinear points lie on the same straight line
05 From the givens to the answer
06 Read a point, a route and its image
07 Check your reasoning
08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Locate a point along a segment

Moving a fraction of the full displacement locates the corresponding internal point.

1

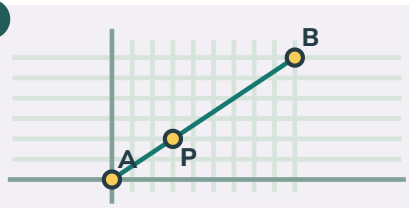


A (-2, -1)
M (2, 2)
B (6, 5)

A(−2, −1) and B(6, 5) share which midpoint?

Half the displacement (8, 6) is (4, 3). Add it to A to get M(2, 2).

2



A (0, 0)
P (3, 2)
B (9, 6)

For A(0, 0) and B(9, 6), where is the 1:2 division point from A?

Travel one of three equal steps, so P = (3, 2).

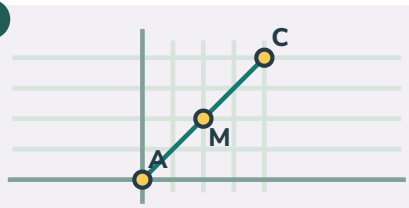
3

A = (−3, 1)
↓
M = (1, 4)
↓
B = (5, 7)

Recover the far endpoint from a midpoint.

From A to M add (4, 3). Add the same vector again to reach B.

4



A (0, 0)
M (2, 2)
C (4, 4)

How can midpoints test a quadrilateral?

Compare both diagonal midpoints. If both bisect each other in a simple quadrilateral, it is a parallelogram.

Midpoint of (0,0) and (8,4)
(4,2).

One quarter of the same trip
(2,1).

CHAPTER PROBLEM

Find the x-coordinate of the midpoint of (−6,1) and (4,7).

-1. Average the two matching coordinates: (-6 + 4)/2 = −1.

MATHS REFERENCE · GRADE 9

Midpoint and coordinate shape checks

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Midpoint	Endpoints (−1,2) and (5,8) give midpoint (2,5) by averaging each coordinate.
Missing endpoint	Midpoint (2,5) and first point (−1,2) give second (2×2 + 1, 2×5 − 2) = (5,8).
Quadrilateral test	If both diagonals of a simple quadrilateral have the same midpoint, they bisect one another, proving a parallelogram.

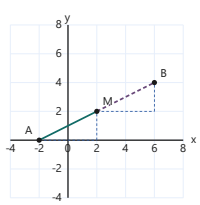
Read the representations

A midpoint makes half of each displacement

To reach the midpoint from A to B, move halfway in the horizontal direction and halfway in the vertical direction.

The two half-displacements reach a point on the segment, not just a point equally far from its endpoints.

That point divides the segment into two equal lengths.



From A = (−2,0) to B = (6,4), each half-displacement is (4,2), giving M = (2,2).

Averaging coordinates is the same half-step

The midpoint x-coordinate is $x_1 + (x_2 - x_1)/2 = (x_1 + x_2)/2$.

Likewise, its y-coordinate is $(y_1 + y_2)/2$. Thus M = $((x_1 + x_2)/2, (y_1 + y_2)/2)$.

Add signed coordinates before halving. The midpoint need not have integer coordinates.



half-turn

Why does a parallelogram map to itself under a half-turn?

Its diagonals bisect each other. Their common midpoint swaps each vertex with its opposite.

THEOREMS AND REPRESENTATIONS

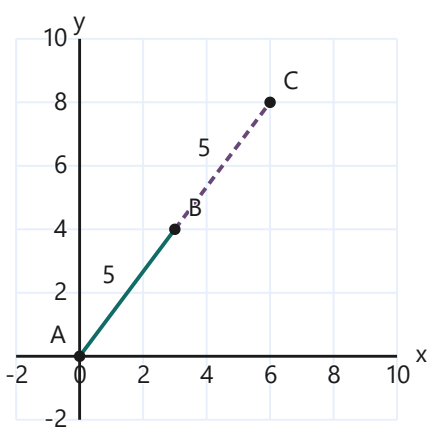
Collinear points lie on the same straight line

Collinear points lie on the same straight line

From (0,0) to (3,4), then to (6,8), the displacement repeats the same direction.

The three pairwise distances are 5, 5 and 10. The longest equals the sum of the other two.

A bent two-part path would be longer than the direct segment. Exact equality places the middle point on it; rounded distances cannot prove that equality.



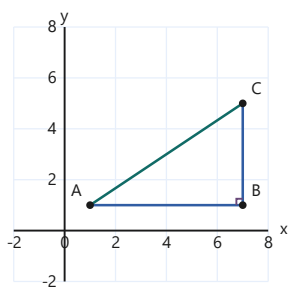
A, B and C lie on one line. AB + BC = AC, with B between the two outer points.

A right-angle claim needs an exact check

Calculate the three squared side lengths of the candidate triangle.

If the largest squared length equals the sum of the other two, the converse of Pythagoras proves a right angle.

Identify the angle opposite the longest side. A picture alone does not justify it.

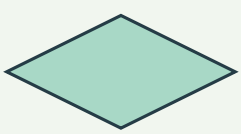


For A = (1,1), B = (7,1), C = (7,5), $AB^2 + BC^2 = 36 + 16 = 52 = AC^2$. The right angle is at B.

Recover a missing endpoint

The midpoint of AB is (2, −1) and A is (−3, 4). From A to the midpoint, move (5, −5). Repeat that move to reach B = (7, −6).

Average the endpoints coordinate by coordinate: (−3 + 7)/2 = 2 and (4 − 6)/2 = −1.



4 equal

Which diagonals split a rhombus into right triangles?

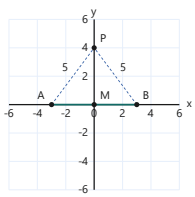
The diagonals meet at right angles and bisect one another.

WORKED EXAMPLES

From the givens to the answer

Does equal distance alone identify a midpoint?

A learner calls (0,4) the midpoint of (−3,0) and (3,0), because it is 5 units from each endpoint.



P is equally distant from A and B, but M is the point halfway along segment AB.

1. Equal distances do not ensure that a point lies on the segment.
2. The segment here lies on y = 0, but (0,4) is off that line.
3. The actual midpoint is (0,0), obtained by averaging the coordinates.

Recover an endpoint from a known midpoint

1. A = (−4,1) and midpoint M = (1,3). Write (−4 + x)/2 = 1 and (1 + y)/2 = 3.
2. The equations give x = 6 and y = 5, so B = (6,5).
3. Check both coordinate averages, not only one.

Is the triangle (0,0), (4,1), (1,5) right-angled?

1. The squared side lengths are $4^2 + 1^2 = 17$, $1^2 + 5^2 = 26$, and $(1 - 4)^2 + (5 - 1)^2 = 25$.
2. The largest is 26, while the other two sum to 42.
3. Since $26 \neq 42$, the triangle is not right-angled.

Check another case

Example 1

Which squared-length comparison proves a triangle is right-angled?

9 + 16 = 25 · 9 + 16 > 20 · 9 + 16 < 36 · All three squares are positive

9 + 16 = 25

The square of the longest side equals the sum of the other squares.

Example 2

Explain a complete midpoint check for a proposed point M between A and B.

Compute the average of the endpoint x-coordinates and compare with M's x.

Compute the average of the endpoint y-coordinates and compare with M's y.

Both must match; equal distances without segment membership are insufficient.



2 mirrors

How many mirror axes does a non-square rectangle have?

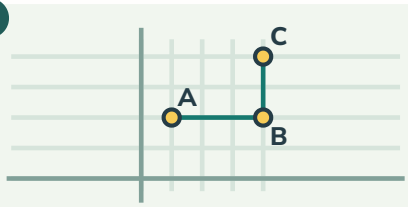
Two, through the midpoints of opposite sides. Its diagonals are not mirror axes.

ILLUSTRATED PROBLEM SHEET

Read a point, a route and its image

Read horizontal position first and vertical position second.

1

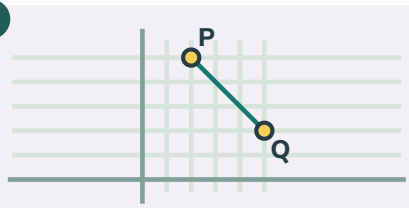


A (1, 2)
B (4, 2)
C (4, 4)

Describe A to C via B.

Go three right, then two up. The total route length is five grid units.

2

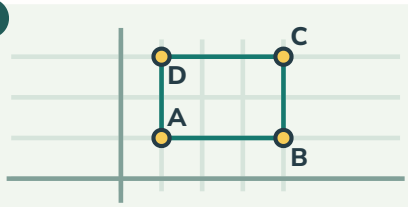


P (2, 5)
Q (5, 2)

Are the points the same if the coordinates swap?

No. Each has its horizontal and vertical distances exchanged.

3

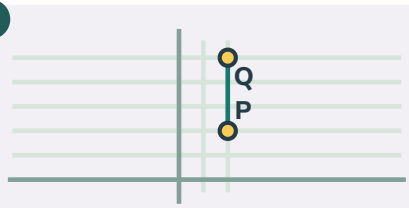


A (1, 1)
B (4, 1)
C (4, 3)
D (1, 3)

How can you recognise this rectangle?

Opposite sides have the same lengths. Horizontal and vertical neighbours meet at right angles.

4



P (2, 2)
Q (2, 5)

Must both coordinates change when a point moves?

No. A vertical move keeps x fixed and adds three to y.

All points on x = 3
a vertical line.

All points on y = 3
a horizontal line.

CHAPTER PROBLEM

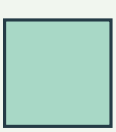
Find the midpoint x-coordinate between (−4,7) and (8,−1).

2. Average the two matching coordinates: (-4 + 8)/2 = 2.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 What is the midpoint of (2,−2) and (8,4)? A) (6,6) B) (5,1) C) (3,3) D) (10,2) 	(5,1) Average each coordinate separately: (2 + 8)/2 = 5 and (−2 + 4)/2 = 1.
02 Does swapping AP:PB from 1:2 to 2:1 keep P fixed?	Only an equal division would stay fixed on reversal. On the segment from (0,0) to (9,6) the new point is (6,4).
03 What point is five right and two down from (1, 4)?	(6, 2). Add five to x and subtract two from y.
04 Find the midpoint y-coordinate between (3,−6) and (9,2).	-2 Average the two matching coordinates: (-6 + 2)/2 = −2.
05 What is the midpoint of (−5,−1) and (1,7)? A) (−2,3) B) (−3,4) C) (−4,6) D) (2,−3)	(−2,3) The coordinate averages are −2 and 3.
06 A has x-coordinate −1. The midpoint has x-coordinate 3. Find the other endpoint's x-coordinate.	7 (−1 + x)/2 = 3 gives x = 7.



4 mirrors

What additional symmetries does a square have?

Both diagonal mirrors also work. A quarter-turn about its centre maps it onto itself.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 A point is equally far from two endpoints. It must be their midpoint only if it also... A) Has positive coordinates B) Lies on their segment C) Has integer coordinates D) Is above both points	Lies on their segment The midpoint is on the segment as well as equally distant from its endpoints.
08 A triangle has squared side lengths 25, 25 and 36. Which conclusion follows? A) It is not right-angled: 25 + 25 ≠ 36 B) It is right-angled because two sides are equal C) It is right-angled because 36 is the largest square D) It is impossible because two squared lengths match	It is not right-angled: 25 + 25 ≠ 36 A right triangle would require its largest squared length to equal the sum of the other two. Here 36 ≠ 50. The side lengths 5, 5 and 6 do form a genuine triangle.
09 Three distinct points have pairwise distances 3, 4 and 7. What does that show?	The longest distance equals the sum of the other two. The points lie on one straight line. The point common to the 3-unit and 4-unit segments lies between the outer points.
10 Explain why a midpoint can have fractional coordinates even when both endpoints have integer coordinates.	Each coordinate is an average of two integers. An odd sum divided by 2 is a half-integer. For example endpoints with x-coordinates 0 and 3 have midpoint x = 1.5.
11 Find the y-coordinate of the midpoint of (−2,−5) and (6,8). 	3/2 Average the two matching coordinates: (-5 + 8)/2 = 3/2.

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