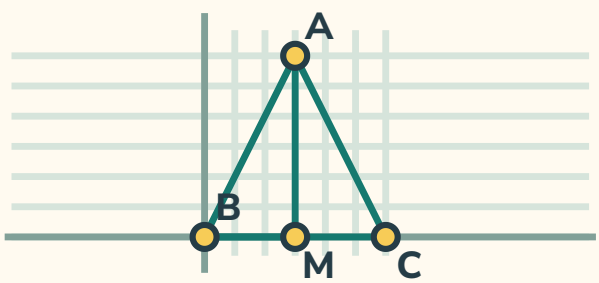
MATHS FIELD GUIDE · GRADE 9

Medians, concurrency and midpoint constructions

OPENING PROBLEM

Why is AM a median?



B (0, 0)
C (6, 0)
A (3, 6)
M (3, 0)

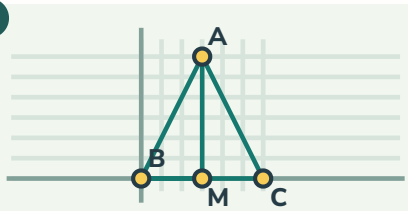
02 Join vertices to side midpoints
03 Rules and worked cases
04 From the givens to the answer
05 Locate a point along a segment
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Join vertices to side midpoints

A median runs from a vertex to the opposite midpoint. The centroid divides each median in a 2:1 ratio from its vertex.

1



B (0, 0)
C (6, 0)
A (3, 6)
M (3, 0)

Why is AM a median?

M(3,0) is exactly halfway from B(0,0) to C(6,0).

2

AM = 12
↓
AG : GM = 2 : 1
↓
AG = 8 and GM = 4

Which piece is longer?

The vertex-to-centroid piece takes two of three equal parts.

3

GM = 5
↓
AG = 10
↓
AM = 15

Can you recover the whole median from the short part?

Yes. Three equal parts of five make fifteen.

4

Vertex	x	y
A	0	6
B	0	0
C	6	0

Where is the centroid?

Average the three x-values and three y-values separately. G = (2,2).

Median 21
vertex-to-centroid length 14.

Vertex-to-centroid length 14
centroid-to-midpoint length 7.

CHAPTER PROBLEM

On median AD, GD = 6 cm and G is the centroid. Find AG in cm.

12. AG is twice GD, so AG = 12 cm.

MATHS REFERENCE · GRADE 9

Medians, concurrency and midpoint constructions

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Build a median	Mark the midpoint of the opposite side, then join it to the vertex. A median need not be perpendicular.
Centroid ratio	On median AM, AG:GM = 2:1. For AM = 15, AG = 10 and GM = 5.
Coordinate check	For vertices (0,0), (6,0), (0,9), averaging coordinates gives centroid (2,3).

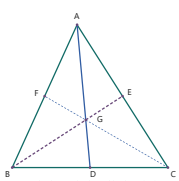
Read the representations

The three medians meet at one point

A median joins a triangle vertex to the midpoint of its opposite side. Let D, E and F be those midpoints.

The medians AD, BE and CF meet at the centroid G. This common intersection is called concurrency.

On each median, the vertex-to-centroid length is twice the centroid-to-midpoint length: AG : GD = 2 : 1. The ratio is measured along the median, not along a triangle side.



The third median has the same intersection

Let the third median CF meet AD at H. Apply the same two-median argument to AD and CF. It gives AH = 2HD.

The point dividing a given segment AD in the vertex-to-midpoint ratio 2 : 1 is unique: it is two-thirds of the way from A to D.

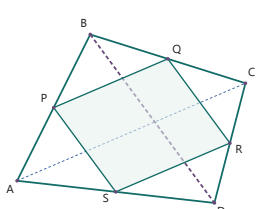
Both G and H have that position on AD, so H = G. All three medians are concurrent.

Side midpoints make a parallelogram inside a quadrilateral

Let P, Q, R and S be the midpoints of AB, BC, CD and DA of a simple quadrilateral ABCD.

In triangles ABC and ADC, the midpoint theorem gives PQ ∥ AC and SR ∥ AC, with both lengths AC/2.

In triangles BCD and BAD, QR and PS are parallel to BD and each equals BD/2. Thus PQRS is a parallelogram, even when ABCD is not.





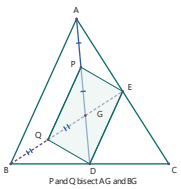
What is the area of this right triangle?

Half of an 8 by 3 rectangle, or 12 cm².

WORKED EXAMPLES

From the givens to the answer

Midpoint segments prove the ratio for two medians



1. Let AD and BE meet at G inside △ABC. Choose P as the midpoint of AG and Q as the midpoint of BG.
2. In △ABG, PQ is parallel to AB and equals AB/2. In △ABC, ED is parallel to AB and equals AB/2. Hence PQDE is a parallelogram.
3. Its diagonals PD and QE meet at G and bisect each other: PG = GD and QG = GE. Since AP = PG and BQ = QG by construction, AG = 2GD and BG = 2GE.

Is the centroid the midpoint of each median?

A learner splits a 24 cm median into two 12 cm pieces at the centroid.

1. The midpoint of the median would give a 1 : 1 split.
2. The centroid gives a 2 : 1 split from the vertex, so the three equal parts are 8 cm each.
3. The vertex-to-centroid length is 16 cm and the centroid-to-side-midpoint length is 8 cm.

Use a different whole median length

1. A median is 33 cm. Its vertex-to-centroid and centroid-to-midpoint parts have ratio 2 : 1.
2. Three equal ratio parts total 33 cm, so one part is 11 cm.
3. The lengths are 22 cm from the vertex and 11 cm from the side midpoint. Their sum is 33 cm.

Connect the ratio to centroid coordinates

1. For A(−2,7), B(−5,−2), C(7,1), the midpoint D of BC is (1,−1/2).
2. G is two-thirds along AD from A. The change from A to D is (3,−15/2), so the two-thirds change is (2,−5). Thus G = (0,2).
3. In general, the same calculation gives G = ((xA + xB + xC)/3, (yA + yB + yC)/3). This averages the three vertex coordinates, not two.

Check another case

Example 1

An outer quadrilateral has diagonals 12 cm and 18 cm. Find the perimeter of its side-midpoint parallelogram in cm.

30

The inner sides are 6 and 9, giving perimeter 2(6 + 9) = 30 cm.

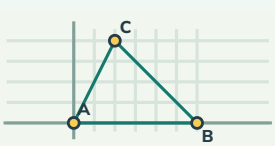
Example 2

Explain why averaging just the endpoints of a median does not locate the centroid.

The two-endpoint average locates the midpoint of that median.

The centroid lies two-thirds of the way from the triangle vertex, not halfway.

Its coordinate formula instead averages all three triangle vertices.



A (0, 0)
B (6, 0)
C (2, 4)

For A(0,0), B(6,0), C(2,4), which height belongs to AB?

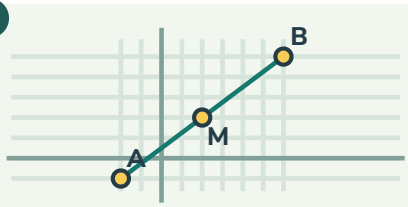
The vertical distance is 4. Area = ½ × 6 × 4 = 12 square units.

ILLUSTRATED PROBLEM SHEET

Locate a point along a segment

Moving a fraction of the full displacement locates the corresponding internal point.

1

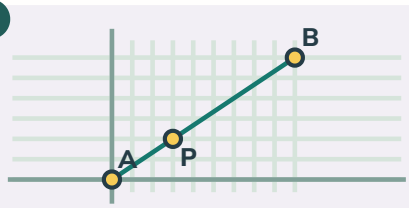


A (-2, -1)
M (2, 2)
B (6, 5)

A(−2, −1) and B(6, 5) share which midpoint?

Half the displacement (8, 6) is (4, 3). Add it to A to get M(2, 2).

2



A (0, 0)
P (3, 2)
B (9, 6)

For A(0, 0) and B(9, 6), where is the 1:2 division point from A?

Travel one of three equal steps, so P = (3, 2).

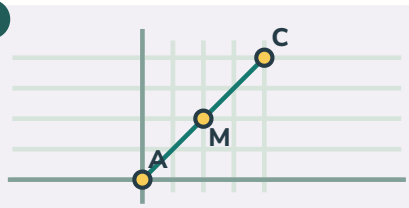
3

A = (−3, 1)
↓
M = (1, 4)
↓
B = (5, 7)

Recover the far endpoint from a midpoint.

From A to M add (4, 3). Add the same vector again to reach B.

4



A (0, 0)
M (2, 2)
C (4, 4)

How can midpoints test a quadrilateral?

Compare both diagonal midpoints. If both bisect each other in a simple quadrilateral, it is a parallelogram.

Midpoint of (0,0) and (8,4)
(4,2).

One quarter of the same trip
(2,1).

CHAPTER PROBLEM

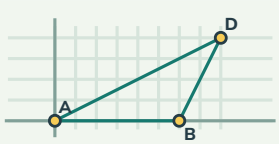
The diagonals of quadrilateral ABCD are 16 cm and 10 cm. Find the perimeter of its side-midpoint parallelogram in cm.

26. Its side lengths are 8 and 5, so its perimeter is 2(8 + 5) = 26 cm.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Must a median also be perpendicular to the opposite side?	No. Equal base segments define a median. Perpendicularity requires extra information.
02 Does swapping AP:PB from 1:2 to 2:1 keep P fixed?	Only an equal division would stay fixed on reversal. On the segment from (0,0) to (9,6) the new point is (6,4).
03 A triangle median is 42 cm. Find the distance from its vertex to the centroid in cm.	28 Two-thirds of 42 is 28 cm.
04 The vertex-to-centroid part of a median is 18 cm. Find its centroid-to-midpoint part in cm.	9 The vertex part is twice the midpoint part, so 18/2 = 9 cm.
05 The centroid-to-midpoint part of a median is 5 cm. Find the whole median in cm.	15 The whole has three such parts: 3 × 5 = 15 cm.
06 What does concurrence of three medians mean? A) They meet at one point B) They all have the same length C) They are mutually perpendicular D) They are triangle sides	They meet at one point Concurrent lines or segments pass through the same point.



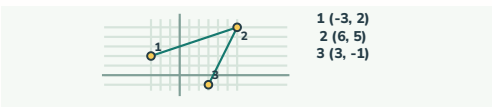
A (0, 0)
B (6, 0)
D (8, 4)

The new apex is D(8,4). Does the area change?

No. The perpendicular foot falls outside AB, but the height to its line is still 4.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 PQ joins midpoints of AB and BC. If AC = 19 cm, find PQ in cm.	19/2 In △ABC, PQ = AC/2 = 19/2 cm.
08 A triangle has vertices (−3,2), (6,5), (3,−1). Find the centroid's x-coordinate. 	2 The x-coordinate is (−3 + 6 + 3)/3 = 2.
09 Explain why a quadrilateral's side-midpoint figure is a parallelogram.	One opposite pair of inner sides is parallel to the same outer diagonal by the triangle midpoint theorem. The other pair is parallel to the other outer diagonal. Both inner opposite side pairs are parallel, satisfying the parallelogram definition.
10 Explain how proving the 2 : 1 ratio for pairs of medians establishes all three meet at one point.	The first two medians meet AD at its unique 2 : 1 division point. Applying the same argument to the first and third medians gives that same division point on AD. Their intersections coincide, so all three pass through one point.
11 A median is 27 cm long. Find its centroid-to-side-midpoint part in cm.	9 One-third of 27 is 9 cm.

Area = 30 cm²
↓
Base = 10 cm
↓
Height = 2 × 30 ÷ 10

Find the height without knowing the sloping sides.

The required perpendicular height is 6 cm.

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