MATHS FIELD GUIDE · GRADE 9

# Linear pairs as intersecting lines

OPENING PROBLEM

## Where do the lines meet?

| x | 0 | 1 | 2 |
|---|---|---|---|
| y = x + 1 | 1 | 2 | 3 |
| y = 5 − x | 5 | 4 | 3 |

02 Decide what the intersection means
03 Rules and worked cases
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05 Move between a rule, a table and a line
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07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

# Decide what the intersection means

Each equation selects a line. A shared point satisfies both equations.

1

| x | 0 | 1 | 2 |
|---|---|---|---|
| y = x + 1 | 1 | 2 | 3 |
| y = 5 − x | 5 | 4 | 3 |

### Where do the lines meet?

At (2, 3). Both output rows give 3 in the same column.

2

$$y = x + 1$$
$$\downarrow$$
$$x + y = 5$$
$$\downarrow$$
$$x + (x + 1) = 5$$

### Substitute to recover the same pair.

2x = 4, so x = 2. Then y = 3.

3

$$x + y = 5$$
$$\downarrow$$
$$-x + y = 1$$
$$\downarrow$$
$$2y = 6$$

### Why add the equations?

The x terms cancel. y = 3, then x = 2.

4

| Line A | Line B | Solutions |
|---|---|---|
| y = 2x + 1 | y = 2x + 4 | none |
| y = 2x + 1 | 2y = 4x + 2 | infinite |

### What changes when the slopes match?

Different intercepts give parallel lines. Matching intercepts give the very same line.

**Same slope, different intercepts**
**no solution.**

**Different slopes**
**exactly one solution.**

CHAPTER PROBLEM

### The lines y = x + 2 and y = 8 − x meet at what x-coordinate?

3. x + 2 = 8 − x gives 2x = 6 and x = 3.

MATHS REFERENCE · GRADE 9

# Linear pairs as intersecting lines

## Rules, conditions and worked cases

| Recognise the case | Mathematical reasoning |
|---|---|
| **Intersection** | $y = 2x$ and $y = 9 - x$ meet when $2x = 9 - x$, at (3,6). |
| **Parallel lines** | $y = 2x$ and $y = 2x + 1$ keep a vertical gap of one, giving no common point. |
| **Coincident lines** | $y = 2x$ and $2y = 4x$ name the same points. Every point on the line is a solution. |

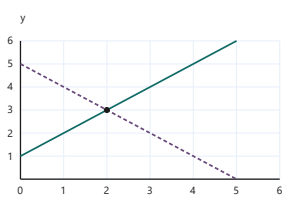
## Read the representations

### A common point satisfies both relationships

Each line represents the real ordered pairs satisfying one equation.

A solution of a pair must belong to both lines at the same time.

When the lines intersect once, the intersection gives the unique common pair.




Solid: $y = x + 1$. Dashed: $y = 5 - x$.

**At (2,3), both relationships are true. This point belongs to both lines.**

### Check coordinates in both equations

Checking one equation only shows that a point is on one line.

For $y = 3x$ and $x + y = 8$, (1,3) fits the first equation but not the second.

Keep x as the first coordinate and y as the second when substituting.

### A graph suggests a pair; exact arithmetic verifies it

Choose two distinct points on each line and plot with labelled scales.

Read the crossing as accurately as the grid permits.

If an intersection lies between grid marks, the graph gives an estimate. Substitution checks an exact proposed pair.

$$x + 7 = 19$$
$$\downarrow$$
$$x = 19 - 7$$
$$\downarrow$$
$$x = 12$$

### Which change keeps both sides balanced?

Subtract seven from each side. Substitution gives $12 + 7 = 19$.

WORKED EXAMPLES

# From the givens to the answer

### Does each equation have a separate final answer?

A learner reports one point from each line as the solution of the pair.

1. The two conditions must hold for the same x and y.
2. A point on only one line does not meet the other condition.
3. Look for a shared point, then check that pair in both equations.

### Check (4,3) for x + y = 7 and x − y = 1

1. Substitute into the sum: 4 + 3 = 7.
2. Substitute into the difference: 4 − 3 = 1.
3. Both are true, so (4,3) is a common solution.

### Plot x + y = 6 and y = x

1. For the first line use (0,6) and (6,0). For the second use (0,0) and (6,6).
2. The lines cross at (3,3).
3. Check: 3 + 3 = 6 and 3 = 3.

## Check another case

### Example 1

Which point is on both x = 2 and y = −3?
(−3,2) · (2,−3) · (2,3) · (0,0)

(2,−3)

The two equations directly fix x and y.

### Example 2

Explain what makes an ordered pair a solution of a system of two equations.

Its x and y values are substituted consistently.

The first equality must be true.

The second equality must also be true for the same pair.

$$3x - 4 = 14 \downarrow 3x = 18 \downarrow x = 6$$

### Which step undoes multiplication?

Dividing both sides by three. Check: 18 − 4 = 14.

ILLUSTRATED PROBLEM SHEET

# Move between a rule, a table and a line

For y = mx + b, b sets the vertical intercept and m is the change in y per unit change in x.

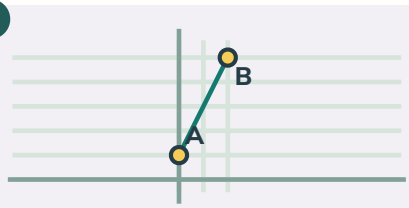
**1**

| x | −1 | 0 | 1 |
|---|---|---|---|
| y = 2x + 1 | −1 | 1 | 3 |

### What stays constant?

Each x-step of one raises y by two. The value at zero is one.

**2**




A (0, 1)
B (2, 5)

### Find rise and run between the points.

Rise 4, run 2. Slope = 4/2 = 2.

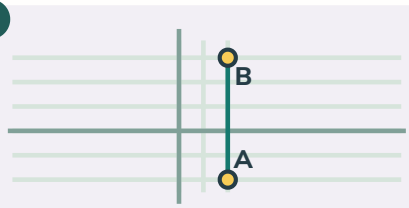
**3**

$$y = 3x - 2$$
$$\downarrow$$
$$10 = 3x - 2$$
$$\downarrow$$
$$x = 4$$

### Which input produces output ten?

Undo subtraction, then divide by three. Check: 3 × 4 − 2 = 10.

**4**




A (2, -2)
B (2, 3)

### Does this vertical line have slope zero?

No. Its run is zero, so rise divided by run is undefined. A horizontal line has slope zero.

**y = 4**
**horizontal, slope zero.**

**x = 4**
**vertical, slope undefined.**

CHAPTER PROBLEM

### For y = 2x and x + y = 12, find the common y-value.

8. x + 2x = 12 gives x = 4 and y = 8.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

# Check your reasoning

| Try the problem | Solution and reasoning |
|---|---|
| **01**<br>Which pair satisfies x + y = 9 and x − y = 3?<br>A) (6,3) B) (3,6) C) (9,3) D) (5,4) | (6,3)<br>6 + 3 = 9 and 6 − 3 = 3. |
| **02**<br>A notebook and pen cost ₹40. Two notebooks plus that pen cost ₹65. Find each price. | Subtract the totals: notebook ₹25. The pen is ₹15. Check both purchases. |
| **03**<br>Which pair satisfies 2x + y = 8 and x + y = 5?<br>A) (3,2) B) (2,3) C) (4,0) D) (1,6) | (3,2)<br>2 × 3 + 2 = 8 and 3 + 2 = 5. |
| **04**<br>Can a rule give two different outputs for the very same input and still be a function? | No. Each allowed input has one output. |
| **05**<br>If a point satisfies only the first equation, it is...<br>A) Automatically the solution<br>B) On both lines<br>C) Not a common solution yet D) The origin | Not a common solution yet<br>The second equation must also hold. |
| **06**<br>Find the common x-value of y = 4 and y = x − 1. | 5<br>4 = x − 1 gives x = 5. |

**2(x + 3) = 2x + 6**
↓
**2x + 6 = 2x + 6**
↓
**0 = 0**

### What does this final equality mean?

Every allowed x satisfies the original equation.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

# Check your reasoning

| Try the problem | Solution and reasoning |
|---|---|
| **07**<br>What does an intersection at (−2,5) mean?<br>A) x = 5 and y = −2<br>B) x = −2 and y = 5 satisfy both equations<br>C) Only y is determined<br>D) The two lines have no common point | x = −2 and y = 5 satisfy both equations<br>The ordered pair gives one common x and y. |
| **08**<br>For x = 3 and 2x + y = 11, find the common y-coordinate. | 5<br>6 + y = 11 gives y = 5. |
| **09**<br>Why is a graph crossing not always an exact numerical measurement? | The drawing has a finite scale and line thickness.<br>A crossing between labelled grid points may only be read approximately.<br>Verify an exact proposed pair by substitution in both equations. |
| **10**<br>Explain why (2,4) is not a solution of y = 2x and x + y = 7. | It satisfies y = 2x because 4 = 2 × 2.<br>But 2 + 4 = 6, which is not 7.<br>The pair fails the second condition and is not a common solution. |
| **11**<br>Find the common x-coordinate of y = x + 4 and y = 10 − x. | 3<br>2x = 6 gives x = 3. |

$$2(x + 3) = 2x + 7$$
$$\downarrow$$
$$6 = 7$$

**What if the variable disappears into a false statement?**

There is no solution. Subtracting the same 2x did not lose one.

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