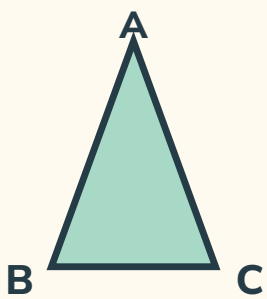
MATHS FIELD GUIDE · GRADE 9

Isosceles triangles and theorem converses

OPENING PROBLEM

If the vertex angle A is 40°, find the base angles.



AB = AC

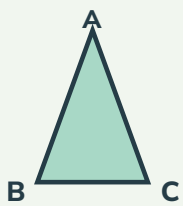
02 Use symmetry, then test the converse
03 Rules and worked cases
04 The converse needs its own argument
05 From the givens to the answer
06 Decide whether the evidence fixes a triangle
07 Check your reasoning
08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Use symmetry, then test the converse

In a triangle equal sides face equal angles. Equal angles also force their opposite sides to match.

1



AB = AC

If the vertex angle A is 40°, find the base angles.

B and C share the remaining 140° equally. Each is 70°.

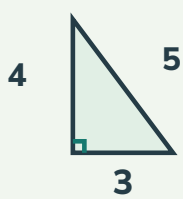
2

∠B = ∠C = 52°
↓
∠A = 76°
↓
AB = AC

Which statement uses the converse?

From equal base angles, deduce their opposite sides AC and AB are equal.

3



An isosceles triangle has sides 5, 5 and base 6. Find its height.

The altitude to the base bisects it. Each half is 3 and the hypotenuse is 5, so the height is 4.

4

M is the base midpoint
↓
AB = AC, BM = CM
↓
AM is common

Why does the median at A bisect the angle?

Triangles ABM and ACM are congruent by SSS. Their corresponding angles at A are equal.

Vertex angle 20°
base angles 80° each.

Each base angle 20°
vertex angle 140°.

CHAPTER PROBLEM

An isosceles triangle has a vertex angle of 36°. Find either base angle in degrees.

72°. The remaining two angles are equal. Subtract 36° from 180°, then divide 144° equally.

MATHS REFERENCE · GRADE 9

Isosceles triangles and theorem converses

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Equal sides	If AB = AC, the opposite angles B and C agree. With ∠A = 38°, both base angles are 71°.
Equal angles	If ∠B = ∠C = 52°, their opposite sides AC and AB are equal. This is the converse.
Find the vertex angle	When both base angles are 68°, the remaining angle is 180° − 136° = 44°.

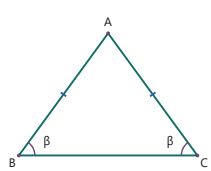
Read the representations

The base angles face the two equal sides

An isosceles triangle has at least two equal sides. Here AB = AC, the base is BC, and the base angles are at B and C.

The base angles are equal because they lie opposite the equal sides. The angle at A is the vertex angle between those sides.

An equilateral triangle is a special isosceles triangle under this inclusive definition.

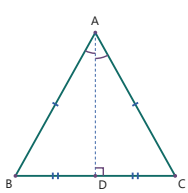


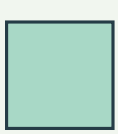
A midpoint creates two congruent halves

A median joins a vertex to the opposite side's midpoint. Choose D as the midpoint of BC and draw median AD. Then BD = DC, AB = AC by the given, and AD is shared.

Thus △ABD ≅ △ACD by SSS, with B ↔ C. Their corresponding base angles are equal, and angle BAD = angle DAC.

An altitude meets the opposite side's line at 90°. The equal angles at D form a linear pair, so each is 90°. Hence AD is a median, angle bisector and altitude.





square

If a quadrilateral is a square, are its diagonals equal?

Yes. They are hypotenuses of congruent right triangles.

THEOREMS AND REPRESENTATIONS

The converse needs its own argument

The converse needs its own argument

The theorem says equal sides imply equal opposite angles. Its converse says equal angles imply equal opposite sides.

If angle B = angle C, compare △ABC with △ACB. The angle pairs at B and C match, and the included side BC equals CB.

ASA gives △ABC ≅ △ACB, with B ↔ C. Consequently AB = AC. The reversed vertex order is deliberate.

The equality must connect opposite parts

If AB = AC, the equal angles are at C and B, not at A and B. Label each side and find the vertex opposite it.

If all three sides are equal, all three angles are equal and sum to 180°, so each is 60°. Conversely, three 60° angles give three equal sides.

An isosceles triangle can be acute, right or obtuse. Equality of two sides does not force a 60° vertex angle.

Work backwards and test the limits

Problem	Derivation
Can an isosceles triangle have angles 35°, 35°, 110°?	Yes. The angles total 180°. The two equal base angles force their opposite sides to be equal. Its remaining angle is obtuse.
Can the vertex angle equal one of the base angles?	Yes. Both base angles already agree, so all three angles would then equal 60°. In this special case the triangle is equilateral.



rectangle

If the diagonals of a quadrilateral are equal, must it be a square?

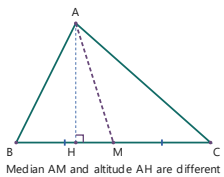
A non-square rectangle is a counterexample. Equal diagonals alone do not force equal sides.

WORKED EXAMPLES

From the givens to the answer

Is every triangle's median also an altitude?

A learner sees the isosceles proof and concludes that any median makes a right angle with the base.



Median AM and altitude AH are different

1. A median joins a vertex to the opposite side's midpoint. An altitude meets the opposite line at 90°.
2. In this scalene triangle M is the midpoint, while H is the perpendicular foot. They are different points.
3. The isosceles proof used AB = AC. Without that given, the SSS comparison of the two halves fails.

Find a different isosceles angle pair

1. A triangle has two equal sides enclosing an angle of 44°. The opposite base angles are equal.
2. Their total is 180 − 44 = 136°, so each is 68°.
3. Check: 44 + 68 + 68 = 180. The equal angles are opposite the equal sides.

Use the converse before solving for a length

1. In △PQR, angles Q and R are equal. The sides opposite them are PR and PQ.
2. By the converse base-angle theorem, PR = PQ. Given PR = 3x + 1 and PQ = 5x − 7, solve 3x + 1 = 5x − 7.
3. This gives x = 4 and both lengths are 13. The angle information justifies the equation.

Check another case

Example 1

In the SSS proof using midpoint D of BC, why is AD = AD?

It is the same shared segment · Every median equals every side · The diagram looks symmetric · All triangles have equal diagonals

It is the same shared segment

Both compared triangles contain the identical segment AD.

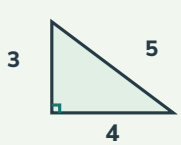
Example 2

Explain the difference between the base-angle theorem and its converse.

The theorem starts with two equal sides and concludes equal opposite angles.

The converse starts with equal angles and concludes equal opposite sides.

Each direction needs justification, and the opposite correspondence must be stated correctly.



Use one diagram to distinguish a theorem from its converse.

A right angle implies $3^2 + 4^2 = 5^2$ for these lengths. The equality also proves the angle is right by the converse of Pythagoras.

ILLUSTRATED PROBLEM SHEET

Decide whether the evidence fixes a triangle

Correspondence records which vertex matches which. The included angle sits between the named sides.

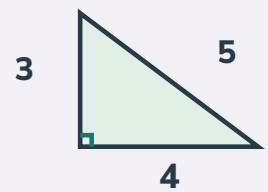
1

Triangle	Side 1	Side 2	Side 3
ABC	3	4	5
PQR	3	4	5

Can all three matching side lengths establish congruence?

Yes, by SSS. Match the vertex opposite each corresponding side.

2



Two triangles each have legs 3 and 4 meeting at 90°. Is that enough?

Yes. Two corresponding sides and their included right angle give SAS.

3

Equal angles: 40°, 60°, 80°

↓

Same shape

↓

Size still free

Do three equal angle pairs prove congruence?

They give similarity. One triangle may be an enlargement of the other.

4

Third side	3	7	11
Other sides	4 and 7	4 and 7	4 and 7
Triangle?	no	yes	no

Why do endpoints fail?

The third side must be strictly between 3 and 11. Equality collapses the triangle into a straight line.

AAA
shape determined, size free.

SSS with fixed lengths
shape and size determined if the triangle exists.

CHAPTER PROBLEM

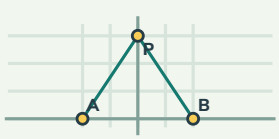
The angle between two equal sides is 102°. Find either base angle in degrees.

39°. The remaining two angles are equal. Subtract 102° from 180°, then divide 78° equally.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 If just two sides match, must the vertex angle be 60°?	No. The base angles must match each other, but their size can vary.
02 In △ABC, angles A and B are equal. Which sides must be equal? A) AB and AC B) BC and AC C) AB and BC D) All three sides without more information	BC and AC The sides opposite A and B are BC and AC.
03 If ABC is congruent to PQR in that order, which side equals AC?	PR. The first and third vertices correspond in both names.
04 An isosceles triangle has base angles 54° each. Find its vertex angle in degrees.	72° 180 − 54 − 54 = 72.
05 If XY = YZ in △XYZ, the equal angles are at... A) X and Z B) X and Y C) Y and Z D) All three vertices necessarily	X and Z XY faces Z and YZ faces X.
06 In △DEF, angles D and F are equal. EF = 4x − 3 and DE = 2x + 7. Find x.	5 EF = DE by the converse, so 4x − 3 = 2x + 7 gives x = 5.



A (-2, 0)
P (0, 3)
B (2, 0)

Why is P(0, 3) equidistant from A(−2, 0) and B(2, 0)?

The perpendicular bisector is the y-axis. Both squared distances equal 4 + 9 = 13.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 A triangle has angles 45°, 45° and 90°. It is... A) Isosceles and right-angled B) Equilateral C) Impossible D) Scalene	Isosceles and right-angled Two equal angles give two equal sides, and one angle is 90°.
08 Which extra given makes median AM in △ABC provably perpendicular to BC? A) AB = AC B) AB is positive C) M is a point on BC without being the midpoint D) Angle A is acute	AB = AC With AM a median, BM = MC. AB = AC and shared AM give SSS, then equal supplementary angles at M.
09 Prove that an equiangular triangle is equilateral.	All three angles are equal. Applying the converse base-angle theorem to pairs of equal angles gives their opposite sides equal. Thus all three sides are equal, which defines an equilateral triangle.
10 Explain why two equal side markings alone do not justify calling a triangle equilateral.	They establish equality of only the marked pair. The third side need not have the same length. For example side lengths 5, 5 and 6 make an isosceles triangle that is not equilateral.
11 An isosceles triangle has base angles of 67°. Find its vertex angle in degrees.	46° 180 − 134 = 46.

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