MATHS FIELD GUIDE · GRADE 9

Geometric sequences

OPENING PROBLEM

What single multiplier fits every step?

2 6 18 54 ?

02 Separate repeated addition from repeated multiplication
03 Rules and worked cases
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05 Count the factors before using a power rule
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07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Separate repeated addition from repeated multiplication

A geometric sequence repeats a factor. An arithmetic sequence repeats a difference.

1

2 6 18 54 ?

What single multiplier fits every step?

Multiply by three. The next term is 162.

2

Stage	0	1	2	3
Pieces	1	3	9	27

If each piece makes three copies, how many after stage four?

81. Four multiplications carry the initial piece to stage four.

3

$$2 \rightarrow -6 \rightarrow 18$$
$$\downarrow$$
$$r = -3$$

Does a negative ratio prevent growth in size?

No. Signs alternate while the magnitudes triple.

4

Stage	0	1	2
Squares	1	8	64
Area left	1	8/9	64/81

Remove the middle ninth from every surviving square. What grows and what shrinks?

The count grows eightfold while total remaining area is multiplied by 8/9 each round.

r = 1
positive first term stays fixed.

r = 1/2
positive terms halve at each step.

CHAPTER PROBLEM

Find the ratio of 6, 18, 54, ...

3. 18/6 = 3 and 54/18 = 3.

MATHS REFERENCE · GRADE 9

Geometric sequences

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Identify ratio	81, 27, 9, 3 has common ratio 1/3. Each term is one third of its predecessor.
Direct rule	For first term 5 and ratio −2, term four is $5(-2)^3 = -40$.
Find the position	For 3,6,12,... solve 3 × 2^(n−1) = 96. Since 96/3 = 32 = 2^5, n = 6.

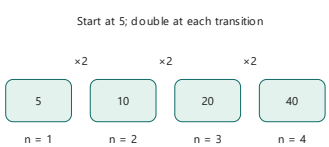
Read the representations

Geometric change repeats a multiplier

A geometric sequence multiplies each term by the same ratio r to obtain the next.

For non-zero terms, find r by dividing a term by its predecessor.

A ratio greater than 1 can model growth. A positive ratio between 0 and 1 can model decay.



Here each term is twice its predecessor. The equal feature is the ratio, not the difference.

The exponent counts the multiplications

Starting with a, the first terms are a, ar, ar^2, ar^3, … .

For non-zero r, position n needs n − 1 multiplications, so $a_n = ar^{n-1}$.

If r = 0, keep the first term a separately; all later terms are zero. Do not use 0^0 here.

A negative ratio alternates the sign

With first term 3 and ratio −2, the terms are 3, −6, 12, −24, … .

Each multiplication changes the sign. Use brackets when raising a negative ratio to a power.

Constant ratio and constant difference describe different sequence rules.

3 7 11 15 ?

What is the next term if the rule is add four?

Nineteen. Each gap is four.

WORKED EXAMPLES

From the givens to the answer

Does doubling add the same amount?

A learner calls 5, 10, 20, 40, ... arithmetic because it grows regularly.

1. The differences are 5, 10 and 20, which are not equal.
2. The ratios are 2, 2 and 2.
3. It is geometric: the multiplier repeats.

Find the sixth term when a = 4 and r = 3

1. $a_6 = 4 \times 3^5$ because five multiplications follow the first term.
2. $3^5 = 243$, so $a_6 = 972$.
3. Check that the exponent is one less than the position.

Find the fourth term of 80, 40, 20, ...

1. The ratio is 1/2.
2. $a_4 = 80 \times (1/2)^3 = 10$.
3. The recursive view confirms 80, 40, 20, 10.

Check another case

Example 1

Derive the geometric nth-term formula for non-zero r.

The first term contains no multiplication by r after a.

The second contains one, and the third two.

Position n therefore contains n − 1 factors of r, giving ar^{n-1}.

Example 2

Find the ratio of 48, 12, 3, ...

1/4

12/48 = 3/12 = 1/4.

Position	1	2	3	4
Term	3	7	11	15

How do you reach term 4 straight from term 1?

Start at 3 and add three jumps of 4. The result is 15.

ILLUSTRATED PROBLEM SHEET

Count the factors before using a power rule

Multiplying equal bases joins factor lists. Dividing cancels matching factors when the base is nonzero.

1

$$2^3 \times 2^2$$
$$\downarrow$$
$$(2 \times 2 \times 2) \times (2 \times 2)$$
$$\downarrow$$
$$2^5 = 32$$

Why add the exponents?

All five factors have the same base.

2

$$3^4 \div 3^2$$
$$\downarrow$$
$$(3 \times 3 \times 3 \times 3) \div 9$$
$$\downarrow$$
$$3^2 = 9$$

Why subtract the exponents?

Two numerator factors cancel with two denominator factors.

3

$$5^0 = 5^3 \div 5^3$$
$$\downarrow$$
$$125 \div 125$$
$$\downarrow$$
$$1$$

What condition matters for a zero exponent?

Use a nonzero base so that the division exists.

4

$$0.0048$$
$$\downarrow$$
$$4.8 \times 10^{-3}$$
$$\downarrow$$
$$4.8 \div 1000$$

How does the minus sign change the scale?

It selects the reciprocal power of ten. The number itself stays positive.

$10^{-2} \times 10^5$
10^3.

$10^{-2} \div 10^5$
10^{-7}.

CHAPTER PROBLEM

Find the fifth term when a = 2 and r = 2.

32. $2 \times 2^4 = 32$.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 A sequence starts at five and doubles. How many doublings take it to 160?	Five: 5, 10, 20, 40, 80, 160. Counting the starting point as a doubling gives one too many.
02 Is $2^3 + 2^2$ equal to 2^5?	No. Addition combines the values 8 and 4 to give 12. The product rule applies to multiplication.
03 Find the fourth term of 7, 21, 63, … .	189 The ratio is 3; $7 \times 3^3 = 189$.
04 Which sequence is geometric? A) 2, 5, 8, 11 B) 3, 6, 12, 24 C) 1, 4, 9, 16 D) 4, 7, 11, 16	3, 6, 12, 24 Every term in 3, 6, 12, 24 is twice the previous term.
05 Find the fifth term when a = 3 and r = −2.	48 $3 \times (-2)^4 = 48$.
06 Find the fourth term when a = 64 and r = 1/2.	8 $64 \times (1/2)^3 = 8$.

Choose a sign from the evidence

A geometric sequence has first term 2 and third term 18. Then $2r^2 = 18$, so r may be 3 or −3.

The missing term is 6 or −6. Both give the same third term. A positive third term alone cannot tell which rule was used.

22 17 12 7 ?

Can the jump be negative?

Yes. Subtract five each time. The next term is 2.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Which exponent belongs in the tenth-term rule ar^{n-1}? A) 8 B) 9 C) 10 D) 11	9 Nine multiplications follow term 1 to reach term 10.
08 Explain why 4, 8, 16, ... is not arithmetic.	The consecutive differences are 4 and 8, so they are not constant. The ratios are both 2. It follows a geometric multiplier rule instead.
09 Explain why a negative geometric ratio can produce a positive term.	Each multiplication by the negative ratio reverses the sign. An even number of negative multipliers has positive product. The starting sign and parity of n − 1 determine the term sign.
10 Find the sixth term when a = 5 and r = 2.	160 $5 \times 2^5 = 160$.
11 Which ratio gives positive values that halve at each step? A) 2 B) −2 C) 1/2 D) −1/2	1/2 Multiplying by positive 1/2 halves without changing the sign.

Input	1	2	3
Output	3	5	7

Which rule doubles then adds one?

Output = 2 × input + 1. Input 6 gives 13.

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