MATHS FIELD GUIDE · GRADE 9

Factoring quadratic expressions

OPENING PROBLEM

Which table entry is the vertical intercept?

x	−3	0	2
p(x)	0	−6	0

02 Connect factors, roots and the constant term
03 Rules and worked cases
04 From the givens to the answer
05 Choose a quadratic method
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Connect factors, roots and the constant term

A product is zero when at least one factor is zero.

1

$$(x - 2)(x + 3)$$
$$\downarrow$$
$$x^2 + x - 6$$

Which inputs give zero?

2 and −3. Each makes a different factor vanish.

2

x	−3	0	2
p(x)	0	−6	0

Which table entry is the vertical intercept?

At x = 0 the output is −6. The two zero outputs locate horizontal intercepts.

3

$$2(x - 2)(x + 3)$$
$$\downarrow$$
$$2x^2 + 2x - 12$$

Does doubling the polynomial change its roots?

No. A nonzero constant factor changes the outputs but preserves the zeros.

4

$$\text{Sum} = -1$$
$$\downarrow$$
$$\text{Product} = -6$$
$$\downarrow$$
$$x^2 + x - 6$$

Reconstruct a monic quadratic from these two root facts.

Use $x^2 - (\text{sum})x + \text{product}$. Its factors are (x − 2) and (x + 3).

$p(x) = (x - 4)^2$
one distinct zero.

$p(x) = (x - 4)(x + 4)$
two distinct zeros.

CHAPTER PROBLEM

In $x^2 + bx + 10 = (x + 2)(x + 5)$, find b.

7. The middle coefficient is 2 + 5 = 7.

MATHS REFERENCE · GRADE 9

Factoring quadratic expressions

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Common factor first	$3x^2 - 12x = 3x(x - 4)$. Both original terms contain 3x.
Sum and product	$x^2 + x - 12 = (x + 4)(x - 3)$ because $4 - 3 = 1$ and $4 \times (-3) = -12$.
Split the middle	$2x^2 + 7x + 3 = 2x^2 + 6x + x + 3 = (2x + 1)(x + 3)$.

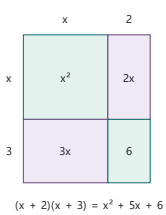
Read the representations

A factored rectangle reveals two brackets

Expanding $(x + p)(x + q)$ gives $x^2 + (p + q)x + pq$.

To factor $x^2 + bx + c$, seek p and q whose sum is b and product is c.

The area model illustrates positive pieces. For signed terms, use algebraic expansion to justify the same structure.



The rectangle $x^2 + 5x + 6$ has sides $x + 2$ and $x + 3$.

The product and sum determine the signs

If pq is positive, p and q have the same sign. Their sum determines whether both are positive or negative.

If pq is negative, their signs are opposite. Compare their magnitudes to match the required sum.

A factorisation is an equivalent expression. It is not a solution for x unless an equation is supplied.

A nonmonic quadratic can use grouping

For $ax^2 + bx + c$, splitting the middle term can create two groups with a shared bracket.

The two split coefficients need product ac and sum b.

Factor each group, then take the common bracket. Check the final product.

×	a	b
a	a^2	ab
b	ab	b^2

Where does 2ab in $(a + b)^2$ come from?

The two rectangular off-diagonal products are both ab.

WORKED EXAMPLES

From the givens to the answer

Can factoring alone tell you x?

A learner factors $x^2 - 5x + 6$ and immediately announces $x = 2$ or 3.

1. The factorisation is $(x - 2)(x - 3)$.
2. No equation has been stated, so x may take any value in the expression.
3. Only an equation such as $x^2 - 5x + 6 = 0$ would allow a zero-product conclusion.

Factor $x^2 - x - 12$

1. Seek two numbers with product -12 and sum -1.
2. The pair is -4 and 3.
3. Therefore $x^2 - x - 12 = (x - 4)(x + 3)$. Expanding confirms the middle term $-x$.

Factor $2x^2 + 7x + 3$

1. The split coefficients have product $2 \times 3 = 6$ and sum 7: use 6 and 1.
2. Write $2x^2 + 6x + x + 3$.
3. Group: $2x(x + 3) + 1(x + 3)$.
4. Take the common bracket: $(2x + 1)(x + 3)$.

Check another case

Example 1

Describe a complete factorisation check.

Expand every proposed factor.

Collect like terms with their signs.

Compare every coefficient and constant with the original expression.

Example 2

Which factors $x^2 + 9x + 20$?

$(x + 4)(x + 5) \cdot (x + 2)(x + 10) \cdot (x - 4)(x - 5) \cdot (x + 1)(x + 20)$

$(x + 4)(x + 5)$

$4 + 5 = 9$ and $4 \times 5 = 20$.

$$(3x - 2)^2 \downarrow (3x)^2 - 2(3x)(2) + 2^2 \downarrow 9x^2 - 12x + 4$$

Which signs change in a squared difference?

The middle product is negative. The constant is positive because $(-2)^2 = 4$.

ILLUSTRATED PROBLEM SHEET

Choose a quadratic method

First write the equation with zero on one side. Factor, complete the square, or use the formula.

1

$$6x^2 + x - 2 = 0$$
$$\downarrow$$
$$(3x + 2)(2x - 1) = 0$$
$$\downarrow$$
$$x = -2/3 \text{ or } 1/2$$

Which products make the middle term?

The cross-products are −3x and +4x, leaving +x.

2

$$x^2 - 6x + 5 = 0$$
$$\downarrow$$
$$(x - 3)^2 = 4$$
$$\downarrow$$
$$x = 1 \text{ or } 5$$

Why do two roots appear?

Both 2 and −2 square to 4, so x − 3 can take either sign.

3

$$x^2 - 2x - 1 = 0$$
$$\downarrow$$
$$D = 8$$
$$\downarrow$$
$$x = 1 \pm \sqrt{2}$$

When is an exact radical useful?

The discriminant is positive and is not a perfect square. The formula preserves both exact roots.

4

$$w(w + 3) = 40$$
$$\downarrow$$
$$(w - 5)(w + 8) = 0$$
$$\downarrow$$
$$w = 5$$

Which algebraic answer fails the model?

A side length cannot be −8. The rectangle is 5 by 8.

$x^2 - 9 = 0$
$x = \pm 3$.

$x^2 + 9 = 0$
no real roots.

CHAPTER PROBLEM

If $x^2 + 2x - 15 = (x + 5)(x + q)$, find q.

-3. The product 5q = −15 gives q = −3, and 5 − 3 = 2.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Which factors $x^2 + 7x + 12$? A) $(x + 3)(x + 4)$ B) $(x + 2)(x + 6)$ C) $(x - 3)(x - 4)$ D) $(x + 1)(x + 12)$	$(x + 3)(x + 4)$ $3 + 4 = 7$ and $3 \times 4 = 12$.
02 Can a nonzero quadratic have three distinct real zeros?	No. Three distinct linear factors would require degree at least three.
03 Which factors $x^2 - 8x + 15$? A) $(x + 3)(x + 5)$ B) $(x - 3)(x - 5)$ C) $(x - 1)(x - 15)$ D) $(x - 5)(x + 3)$	$(x - 3)(x - 5)$ $-3 + (-5) = -8$ and $(-3)(-5) = 15$.
04 What happens to $x^2 - 6x + k = 0$ at $k = 9$?	It becomes $(x - 3)^2 = 0$. The two root values coincide at 3.
05 Which factors $3x^2 + 10x + 3$? A) $(3x + 1)(x + 3)$ B) $(3x + 3)(x + 1)$ C) $(3x - 1)(x - 3)$ D) $(x + 3)^2$	$(3x + 1)(x + 3)$ The expansion is $3x^2 + 9x + x + 3$.
06 Find the coefficient of x when $(2x - 3)(x + 4)$ is expanded.	5 The mixed terms are $8x$ and $-3x$, giving $5x$.

$49x^2 - 16$
↓
$(7x)^2 - 4^2$
↓
$(7x - 4)(7x + 4)$

Why is there no middle term?

The cross-products $+28x$ and $-28x$ cancel.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Which first step factors $4x^2 + 12x$ fully? A) Take common factor 4x B) Take common factor 12 C) Declare x = 0 D) Use $x^2 + 12^2$	Take common factor 4x $4x^2 + 12x = 4x(x + 3)$.
08 Why is finding two numbers with the correct product alone insufficient?	Their sum must also match the middle coefficient. For example, the pair 2 and 6 has product 12, but its sum is 8. A different middle term gives a different quadratic.
09 Explain why factorising $x^2 + x - 6$ does not by itself impose a value of x.	It rewrites the expression as $(x + 3)(x - 2)$. Both forms have the same value for any real x. Without an equation, there is no condition to solve.
10 Which factors $x^2 + 3x - 10$? A) $(x + 5)(x - 2)$ B) $(x - 5)(x + 2)$ C) $(x + 10)(x - 1)$ D) $(x - 5)(x - 2)$	$(x + 5)(x - 2)$ $5 - 2 = 3$ and $5(-2) = -10$.
11 Find b if $(3x + 2)(x - 1) = 3x^2 + bx - 2$.	-1 The mixed terms are $-3x$ and $2x$, giving $-x$.

×	x	−3
x	x^2	−3x
5	5x	−15

Collect the products of $(x + 5)(x - 3)$.

The middle terms combine to 2x, giving $x^2 + 2x - 15$.

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