MATHS FIELD GUIDE · GRADE 9

Explicit and recursive sequence rules

OPENING PROBLEM

What is the next term if the rule is add four?

3 7 11 15 ?

02 Track the jump and the starting point
03 Rules and worked cases
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05 Separate repeated addition from repeated multiplication
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Track the jump and the starting point

A position counts terms. From the first to the nth term there are n − 1 gaps.

1

3 7 11 15 ?

What is the next term if the rule is add four?

Nineteen. Each gap is four.

2

Position	1	2	3	4
Term	3	7	11	15

How do you reach term 4 straight from term 1?

Start at 3 and add three jumps of 4. The result is 15.

3

22 17 12 7 ?

Can the jump be negative?

Yes. Subtract five each time. The next term is 2.

4

Input	1	2	3
Output	3	5	7

Which rule doubles then adds one?

Output = 2 × input + 1. Input 6 gives 13.

Rule “add six”, start two
term four is 20.

Rule “multiply by six”, start two
term four is 432.

CHAPTER PROBLEM

For $a_n = 3n + 1$, find a_4.

13. $3 \times 4 + 1 = 13$.

MATHS REFERENCE · GRADE 9

Explicit and recursive sequence rules

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Recursive rule	Start at 7 and add 4 each time. The first four terms are 7, 11, 15, 19.
Direct rule	Term n is 7 + 4(n − 1). Term 20 is 83 without generating 19 earlier terms.
Test the indexing	7 + 4n gives 11 at n = 1. Use it only if the starting 7 is indexed by zero.

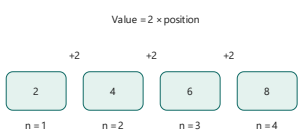
Read the representations

A sequence assigns a value to each position

Positions usually start at n = 1. The symbol a_n means the value at position n.

For 2, 4, 6, 8, ..., position 3 has value 6. The position and value are different numbers.

An explicit rule such as $a_n = 2n$ calculates a term directly from its position.



Each position labels one term. The input is n; the output is a_n.

A recursive rule needs a starting point

A recursive rule tells how to obtain a later term from earlier terms.

The same even-number sequence can be specified by $a_1 = 2$ and $a_n = a_{n-1} + 2$ for $n \geq 2$.

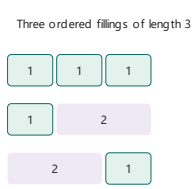
Without the initial term, “add 2” describes many possible sequences.

Some rules need two earlier terms

For the Virahanka pattern used here, $V_1 = 1$, $V_2 = 2$, and $V_n = V_{n-1} + V_{n-2}$ for $n \geq 3$.

The terms count ways to fill n units using pieces of length 1 or 2.

A filling ends with either a length-1 piece or a length-2 piece, giving the two smaller counts to add.



For length 3, the fillings are 1+1+1, 1+2 and 2+1. The order of pieces matters.

7 **11** **15** **19** **23**

What do the four gaps show?

The common difference is 4. Reaching term 5 takes four jumps.

WORKED EXAMPLES

From the givens to the answer

Can “add 3” determine the first term?

A learner claims that “add 3 each time” uniquely means 3, 6, 9, … .

1. It could also mean 1, 4, 7, … or 10, 13, 16, … .
2. The update rule alone does not give the initial value.
3. Supply both a starting value and the recurrence.

Generate $a_1 = 5$, $a_n = 2a_{n-1} - 1$

1. $a_2 = 2 \times 5 - 1 = 9$.
2. $a_3 = 2 \times 9 - 1 = 17$.
3. $a_4 = 2 \times 17 - 1 = 33$. Each calculation uses the previous output.

Use an explicit rule $b_n = n^2 + 1$

1. To find b_5, substitute position n = 5 directly.
2. $b_5 = 25 + 1 = 26$.
3. The earlier four terms are not needed for this calculation.

Check another case

Example 1

Which is the third term of 8, 11, 14, 17, …?

3 · 8 · 14 · 17

14

The value at position 3 is 14.

Example 2

Explain why seeing a few terms does not prove a unique rule for all later terms.

Different rules can match the same finite list.

A stated general rule or defining context is needed to determine later terms.

Checking the visible pattern is evidence, not uniqueness proof.

$a_1 = 7$ and $d = 4$

↓

$a_{12} = 7 + 11 \times 4$

↓

51

Why use eleven instead of twelve?

The first term is already present before making any jumps.

ILLUSTRATED PROBLEM SHEET

Separate repeated addition from repeated multiplication

A geometric sequence repeats a factor. An arithmetic sequence repeats a difference.

1

2 6 18 54 ?

What single multiplier fits every step?

Multiply by three. The next term is 162.

2

Stage	0	1	2	3
Pieces	1	3	9	27

If each piece makes three copies, how many after stage four?

81. Four multiplications carry the initial piece to stage four.

3

$$2 \rightarrow -6 \rightarrow 18$$
$$\downarrow$$
$$r = -3$$

Does a negative ratio prevent growth in size?

No. Signs alternate while the magnitudes triple.

4

Stage	0	1	2
Squares	1	8	64
Area left	1	8/9	64/81

Remove the middle ninth from every surviving square. What grows and what shrinks?

The count grows eightfold while total remaining area is multiplied by 8/9 each round.

$r = 1$
positive first term stays fixed.

$r = 1/2$
positive terms halve at each step.

CHAPTER PROBLEM

For $a_1 = 4$ and $a_n = a_{n-1} + 5$, find a_3.

14. The first three terms are 4, 9, 14.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Do the first two numbers 2, 4 fix the whole sequence?	No. Adding two gives 6 next. Doubling gives 8 next. A rule is needed.
02 A sequence starts at five and doubles. How many doublings take it to 160?	Five: 5, 10, 20, 40, 80, 160. Counting the starting point as a doubling gives one too many.
03 In $a_7 = 23$, what does 7 represent? A) The term value B) The position C) The common difference D) The number of terms always present	The position The subscript identifies the position.
04 For $b_n = 4n - 2$, find b_6.	22 $4 \times 6 - 2 = 22$.
05 Start at 3 and double each term. Find the fourth term.	24 3, 6, 12, 24.
06 Which specification completely defines a recursive sequence? A) Add 4 B) Start at 7 and add 4 each time C) The numbers increase D) The next term is larger	Start at 7 and add 4 each time It supplies both an initial term and an unambiguous update.

Catch the indexing error

A sequence starts 4, 9, 14. The instruction "start at 4 and add 5" gives $a_n = 4 + 5(n - 1)$ for n starting at 1.

Trying $4 + 5n$ gives 9 for the first term. Test the first index before using the rule on a distant term.

$$7 + 4(n - 1) = 39$$
$$\downarrow$$
$$4(n - 1) = 32$$
$$\downarrow$$
$$n = 9$$

Does 39 occur?

Yes. The position is a positive integer. The adjacent terms are 35 and 43.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Using $V_1 = 1$, $V_2 = 2$ and addition of the preceding two terms, find V_6.	13 The terms are 1, 2, 3, 5, 8, 13.
08 For $a_1 = 10$ and $a_n = a_{n-1} - 3$, find a_4.	1 10, 7, 4, 1.
09 Explain how explicit and recursive rules differ.	An explicit rule uses the position to give a value directly. A recursive rule uses previous term values. A recursive definition also needs enough initial values to begin.
10 Why are both V_1 and V_2 needed when each new Virahanka term adds the preceding two?	The update needs two earlier values. To compute V_3 both V_2 and V_1 must already be known. One seed alone leaves the other starting value unspecified.
11 For $c_n = n^2 - 1$, find c_4.	15 $4^2 - 1 = 15$.

Position	3	8
Term	15	35

Recover the difference without knowing term 1.

Five gaps add 20, so d = 4. Step back twice from 15 to get 7.

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