MATHS FIELD GUIDE · GRADE 9

Experimental probability and expected frequency

OPENING PROBLEM

Which trial set landed closer to half heads?

Heads	Tosses	Share
3	4	3/4
5	10	1/2

02 Compare a model with a record
03 Rules and worked cases
04 Expected count is trial count times probability
05 From the givens to the answer
06 Read an experiment before predicting
07 Check your reasoning
08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Compare a model with a record

Relative frequency = event count ÷ trials. A short record can differ from a theoretical chance.

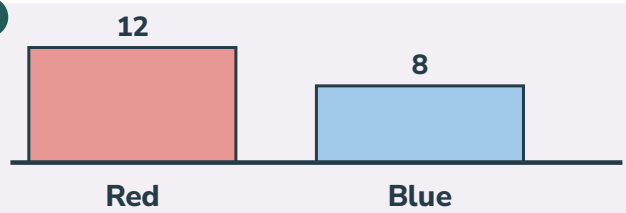
1

Heads	Tosses	Share
3	4	3/4
5	10	1/2

Which trial set landed closer to half heads?

The second. The first did not disprove a fair-coin model.

2



Estimate the chance of red from these twenty draws.

12/20 = 3/5. Record whether each counter was replaced before comparing trials.

3

Set	Hits	Tries
A	3	5
B	9	15

Combine the evidence.

Twelve hits in twenty tries gives 3/5. Pool counts before dividing.

4

Chance ¼
↓
80 trials
↓
80 × ¼ = 20

Does an expectation of twenty guarantee twenty hits?

No. Actual totals vary even when the chance stays fixed.

6 hits in 8 trials
75%.

6 hits in 12 trials
50%.

CHAPTER PROBLEM

An event occurred 21 times in 60 trials. Find its relative frequency.

7/20. 21/60 simplifies to 7/20 = 0.35.

MATHS REFERENCE · GRADE 9

Experimental probability and expected frequency

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Estimate from observations	18 successes in 60 give relative frequency 0.3.
Project to a new count	At estimated rate 0.3, 250 comparable trials predict about 75 successes.
Combine batches	Add successes and trials before dividing. 18/60 and 10/40 together give 28/100, not an unweighted average.

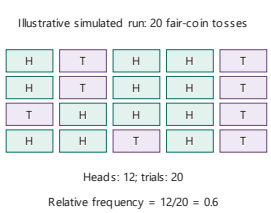
Read the representations

An experiment supplies an empirical estimate

Record the event count and the number of trials. Relative frequency = event count ÷ trial count.

The displayed sequence is an illustrative computer simulation of fair-coin tosses. It is not a record of physical classroom tosses. Count its H results and compare that frequency with the model probability 1/2.

The model probability describes the assumed process. A finite observed frequency can differ from it without proving that the model is wrong.

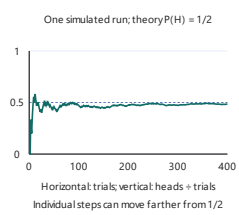


More trials can reveal a stable long-run pattern

The graph tracks cumulative heads divided by cumulative trials in a separate simulated fair-coin run. Its horizontal axis uses the actual number of tosses.

Early results fluctuate strongly. With repeated independent trials under a stable model, relative frequency tends to concentrate near the underlying probability as the trial count grows.

This is a long-run tendency. The graph need not move closer at every step, and no finite sample size guarantees exact agreement.



Question	Population
School travel	all pupils
Club travel	club members

Who is missing from a cycling-club survey of school transport?

Pupils outside the club. They may use transport differently.

THEOREMS AND REPRESENTATIONS

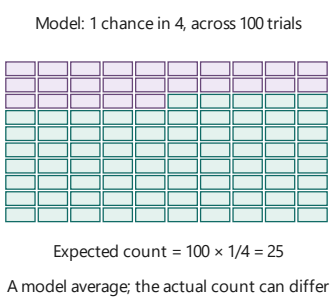
Expected count is trial count times probability

Expected count is trial count times probability

If an event has probability p on each of n comparable trials, its expected count is np.

In the illustration, 100 trials with event probability 1/4 have expected count 25. The coloured cells show the model share, rather than a guaranteed set of observed successes.

Across many repetitions of a comparable 100-trial study, average event counts tend towards 25. One study might give a count above or below that expectation.



A zero observed count does not prove impossibility

If an event has not occurred in a finite trial record, its observed relative frequency is zero.

The event could still be possible but rare, or the observation process could miss it. Distinguish “not observed in these trials” from “excluded by the model”.

Likewise, a repeated event in every observed trial does not by itself prove that the event is certain in all future trials.

Pool trials by adding counts and denominators

To combine compatible records, add their event counts and add their trial counts, then divide.

Use trial counts as weights. A simple average agrees with the pooled frequency when the counts are equal or the two frequencies are equal. Deliberately weighting groups equally describes a different summary.

Check that both records use the same event definition and a comparable process. A changing process may make a single pooled prediction misleading.

Group	Records	Cycle
A	20	8
B	10	6

Pool the observations. What fraction cycle?

14/30 = 7/15. Averaging 40% and 60% would give both groups equal weight.

WORKED EXAMPLES

From the givens to the answer

Use a recorded frequency for a fresh prediction

1. A fictional quality-check record contains 18 faulty items in 300 comparable checks. The estimated fault probability is 18/300 = 0.06.
2. For 450 future comparable checks, the estimated expected fault count is 450 × 0.06 = 27.
3. The estimate assumes that the process and checking method remain similar. It does not guarantee exactly 27 faulty items.

Does a fair coin owe a head after several tails?

A learner says a fair coin must produce a head next because tails has appeared repeatedly.

1. In the independent fair-coin model, the next toss still has head probability 1/2.
2. Earlier outcomes do not change that next-toss probability.
3. Long-run balance does not require a compensating result on a particular next trial.

Check another case

Example 1

A model predicts 40 successes, but a study observes 37. Explain how to report this.

State the model expectation of 40 and the actual observed count of 37 separately.

The finite observed count can differ because of chance variation and does not have to equal expectation.

Check model assumptions and further comparable evidence before concluding that a discrepancy reveals bias or failure.

Example 2

An event occurs 27 times in 90 trials. Find its relative frequency.

3/10

27/90 = 3/10.

Day	Monday	Tuesday
Rain?	yes	no
Walkers	8	20

Does the difference prove rain caused the change?

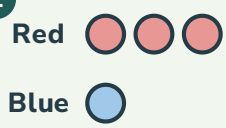
It is consistent with that idea, but other day-to-day differences may explain it.

ILLUSTRATED PROBLEM SHEET

Read an experiment before predicting

Identify one trial, its possible results and what counts as success.

1



Draw one counter without looking. Which result is possible but less likely?

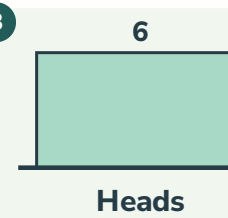
Blue. One of the four equal-chance counters is blue.

2

What changed when the last blue was taken away?

A blue draw became impossible. A red draw is certain.

3



A child records ten coin tosses. Did both outcomes occur?

Yes. Heads occurred more often in this record.

4

Try	1	2	3	4
Colour	R	B	R	R

Return each counter and mix before drawing again. Must B come next?

No. A short run of reds does not force the next colour.

An all-red bag
a red draw is certain.

Add one blue
a red draw is possible but no longer certain.

CHAPTER PROBLEM

A model assigns probability 0.3 to an event on each of 250 trials. Find the expected count.

75. 250 × 0.3 = 75.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 One group gets 1 hit in 2 tries, another gets 9 in 10. Is the pooled rate 70%?	No. Ten hits in twelve tries gives 5/6. The groups had different sizes.
02 Can you fairly compare two guessing games if one player gets twice as many turns?	Compare successes per turn or give both the same number of turns.
03 An event occurs 32 times in 80 trials. Find its relative frequency.	2/5 32/80 = 2/5.
04 A model gives probability 3/8 for an event. Find its expected count in 160 trials.	60 160 × 3/8 = 60.
05 A fictional process record shows 14 faults in 200 checks. Under the same estimated rate, find the expected faults in 500 future checks.	35 14/200 × 500 = 35.
06 Two comparable records show 5 successes in 20 trials and 19 in 60. Find the pooled relative frequency.	3/10 (5 + 19)/(20 + 60) = 24/80 = 3/10.

Measure	Before	After
Total pupils	20	40
Cyclists	10	16

Which rose: the headcount or share cycling?

The count rose by six. The share fell from 50% to 40%.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 A probability model gives expected count 12.5 in a fixed set of trials. Which statement is valid? A) The model is impossible because the expectation is fractional B) Expectation can be fractional even though observed counts are whole C) Exactly 12.5 successes must be observed D) The probability must exceed 1	Expectation can be fractional even though observed counts are whole Expected count is a model average and need not be an integer.
08 A rare event occurs zero times in 40 trials. What follows directly? A) The next trial must produce it B) Its observed frequency in this record is zero C) Its true probability must be zero D) It can never occur in the future	Its observed frequency in this record is zero A finite absence does not establish impossibility.
09 A relative-frequency graph moves away from 1/2 between two adjacent points. Does that contradict a fair-coin model?	No. Individual trials change both the cumulative count and denominator. A long-run tendency towards the model probability does not require every step to be closer. Assess the process and a substantial record rather than rejecting the model from a single upward or downward move.
10 An estimate from last year's checks is used after the manufacturing process changes. What should be checked?	The old record estimates the earlier process under its event definition and checking method. A process change may alter the probability, making old and new checks non-comparable. Collect and evaluate suitable recent evidence before treating the old rate as a reliable future prediction.
11 An event has probability 0.18 per trial. Find its expected count in 350 trials.	63 $350 \times 0.18 = 63$.

BANANALEARN LEARNING LIBRARY

Where will your maths take you next?

GRADE 9 MATHS

Choose your next problem.

Explore more illustrated guides and practise a related topic.

Browse Grade 9 guides →

Keep exploring

GRADE 9 · TOPIC GUIDE

Counting outcomes without double-counting

Open guide →

GRADE 9 · TOPIC GUIDE

Independent combined events

Open guide →

Visit bananalearn.com →