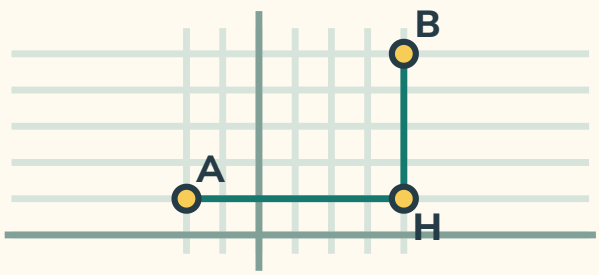
MATHS FIELD GUIDE · GRADE 9

Distance in the coordinate plane

OPENING PROBLEM

Find the direct distance from A(−2, 1) to B(4, 5).



A (-2, 1)
H (4, 1)
B (4, 5)

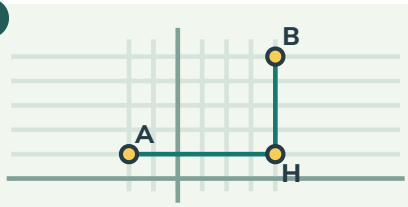
02 Build the right triangle between two points
03 Rules and worked cases
04 From the givens to the answer
05 Read a point, a route and its image
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Build the right triangle between two points

The horizontal and vertical displacements are perpendicular. Their squares add to the squared direct distance.

1

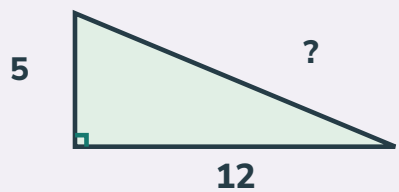


A (-2, 1)
H (4, 1)
B (4, 5)

Find the direct distance from A(−2, 1) to B(4, 5).

The legs are 6 and 4. AB = √52 = 2√13. The broken route via H is 10 units.

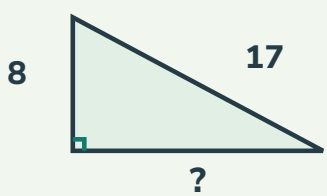
2



A right triangle has legs 5 and 12. How long is its hypotenuse?

$5^2 + 12^2 = 169$, so the longest side is 13.

3



The hypotenuse is 17 and one leg is 8. Recover the other leg.

Subtract squares: 289 − 64 = 225. The missing positive length is 15.

4

$$7^2 + 24^2$$
$$\downarrow$$
$$49 + 576 = 625$$
$$\downarrow$$
$$25^2$$

Can sides 7, 24 and 25 make a right triangle?

Yes. The converse places the right angle opposite the longest side, 25.

Coordinate changes (0,7)
distance 7.

Coordinate changes (−3,4)
distance 5.

CHAPTER PROBLEM

Find the distance from (0,0) to (3,4).

5. The coordinate changes are 3 and 4. Their squares add to 25. Taking the positive square root gives the distance.

MATHS REFERENCE · GRADE 9

Distance in the coordinate plane

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Horizontal or vertical	(−4,2) to (3,2) has length seven, the absolute change in x.
Diagonal	(−2,1) to (3,13) gives legs 5 and 12. Distance $\sqrt{169} = 13$.
Compare squared lengths	A segment with changes 4 and 7 has squared length 65. It is longer than an 8-unit segment since $65 > 64$.

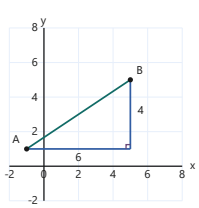
Read the representations

A right triangle connects the coordinate changes

Horizontal change is the difference between x-coordinates. Vertical change is the difference between y-coordinates.

The horizontal and vertical directions are perpendicular, so they form the legs of a right triangle. The segment joining the points is its hypotenuse.

For a physical distance, both axes must measure perpendicular directions in the same length unit.



A = (−1,1), B = (5,5). The horizontal leg is 6 units and the vertical leg is 4 units.

Pythagoras gives the distance formula

For $A(x_1,y_1)$ and $B(x_2,y_2)$, the squared distance is $AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$.

Therefore $AB = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$.

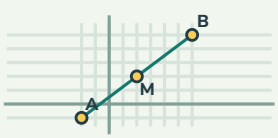
Signed differences can be negative; their squares are non-negative. Reversing the point order gives the same distance.

A distance and its square have different meanings

If $AB^2 = 20$, the length is $\sqrt{20}$, not 20. Keep the radical for an exact answer.

A decimal can estimate a root, but use ≈ when rounding.

A distance is non-negative. It is zero exactly when the two points coincide.



A (-2, -1)
M (2, 2)
B (6, 5)

A(−2, −1) and B(6, 5) share which midpoint?

Half the displacement (8, 6) is (4, 3). Add it to A to get M(2, 2).

WORKED EXAMPLES

From the givens to the answer

Is the diagonal the sum of the coordinate changes?

A learner adds a horizontal change of 3 and a vertical change of 4 to report a straight-line distance of 7.

1. That addition measures a two-part right-angle route.
2. The direct segment has squared length $3^2 + 4^2 = 25$.
3. Its length is 5. The route and the direct distance are different quantities.

Find the distance from (−2,−1) to (4,7)

1. The horizontal change is $4 - (-2) = 6$. The vertical change is $7 - (-1) = 8$.
2. $AB^2 = 6^2 + 8^2 = 36 + 64 = 100$.
3. The distance is $\sqrt{100} = 10$ coordinate units.

Check a horizontal and a vertical special case

1. Between (−5,3) and (2,3), the vertical change is 0, so the length is $\sqrt{(7^2)} = 7$.
2. Between (4,−3) and (4,6), the horizontal change is 0, so the length is $\sqrt{(9^2)} = 9$.
3. The formula agrees with counting the one changing coordinate.

Check another case

Example 1

Explain the difference in units between a length and its squared length.

If coordinate lengths are measured in cm, the distance is in cm.

Squaring makes the units cm^2.

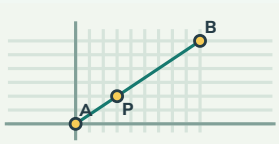
Taking the square root returns the length unit, so a squared total should not be reported as the distance.

Example 2

Find the distance from (0,0) to (8,15).

17

$64 + 225 = 289$ and $\sqrt{289} = 17$.



A (0, 0)
P (3, 2)
B (9, 6)

For A(0, 0) and B(9, 6), where is the 1:2 division point from A?

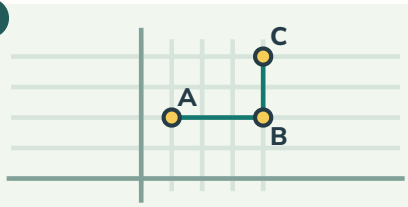
Travel one of three equal steps, so P = (3, 2).

ILLUSTRATED PROBLEM SHEET

Read a point, a route and its image

Read horizontal position first and vertical position second.

1

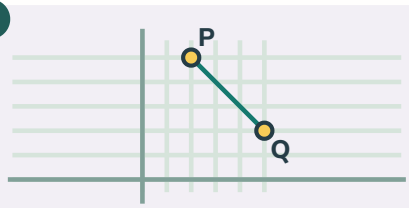


A (1, 2)
B (4, 2)
C (4, 4)

Describe A to C via B.

Go three right, then two up. The total route length is five grid units.

2

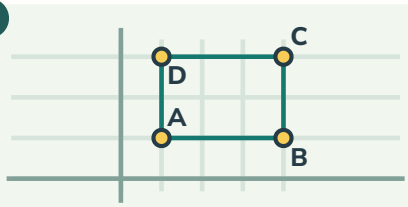


P (2, 5)
Q (5, 2)

Are the points the same if the coordinates swap?

No. Each has its horizontal and vertical distances exchanged.

3

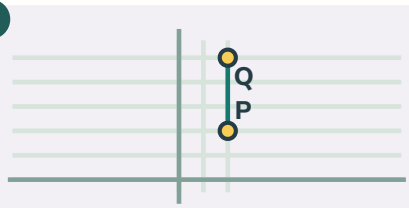


A (1, 1)
B (4, 1)
C (4, 3)
D (1, 3)

How can you recognise this rectangle?

Opposite sides have the same lengths. Horizontal and vertical neighbours meet at right angles.

4



P (2, 2)
Q (2, 5)

Must both coordinates change when a point moves?

No. A vertical move keeps x fixed and adds three to y.

All points on x = 3
a vertical line.

All points on y = 3
a horizontal line.

CHAPTER PROBLEM

Find the distance between (−3,2) and (2,14).

13. The coordinate changes are 5 and 12. Their squares add to 169. Taking the positive square root gives the distance.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Is a route 3 units east and 4 north 7 units from its start?	The route length is 7. The straight-line distance is 5.
02 Which expression is the distance between $A(x_1,y_1)$ and $B(x_2,y_2)$? A) $(x_2 - x_1) + (y_2 - y_1)$ B) $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$ C) $(x_2 - x_1)^2 - (y_2 - y_1)^2$ D) $\sqrt{(x_1^2 + y_1^2)}$	$\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$ Pythagoras uses both coordinate differences as perpendicular legs.
03 What point is five right and two down from (1, 4)?	(6, 2). Add five to x and subtract two from y.
04 Find the squared distance between (1,−2) and (4,5). 	58 The changes are 3 and 7, so the squared distance is 9 + 49 = 58.
05 What is the exact distance from (0,0) to (2,3)? A) 5 B) 13 C) $\sqrt{13}$ D) $\sqrt{5}$	$\sqrt{13}$ The squared length is 4 + 9 = 13.
06 Find the distance from (−4,−1) to (5,−1).	9 The coordinate changes are 9 and 0. Their squares add to 81. Taking the positive square root gives the distance.

A = (−3, 1)
↓
M = (1, 4)
↓
B = (5, 7)

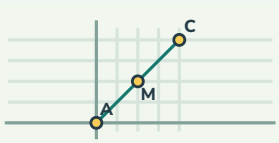
Recover the far endpoint from a midpoint.

From A to M add (4, 3). Add the same vector again to reach B.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Reversing the order of two endpoints changes... A) The distance to its negative B) Neither the distance nor its square C) The length into the sum of coordinates D) The endpoints into the origin	Neither the distance nor its square Both signed differences reverse, but their squares remain the same.
08 Find the distance between (−2,4) and (−2,4). 	0 The coordinate changes are 0 and 0. Their squares add to 0. Taking the positive square root gives the distance.
09 Why should this physical distance formula not be applied directly to a time-temperature graph?	Its coordinates measure different types of quantities. A time difference and a temperature difference are not perpendicular length components in one common unit. Their squared sum would not represent the intended physical length.
10 Explain why subtracting coordinates in the opposite order does not change the distance.	Each difference is replaced by its negative. A number and its negative have the same square. The squared sum and non-negative square root therefore stay unchanged.
11 If the squared distance is 29, what is the exact length? A) 29 B) 58 C) √29 D) 29^2	√29 Distance is the non-negative square root of its square.



A (0, 0)
M (2, 2)
C (4, 4)

How can midpoints test a quadrilateral?

Compare both diagonal midpoints. If both bisect each other in a simple quadrilateral, it is a parallelogram.

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