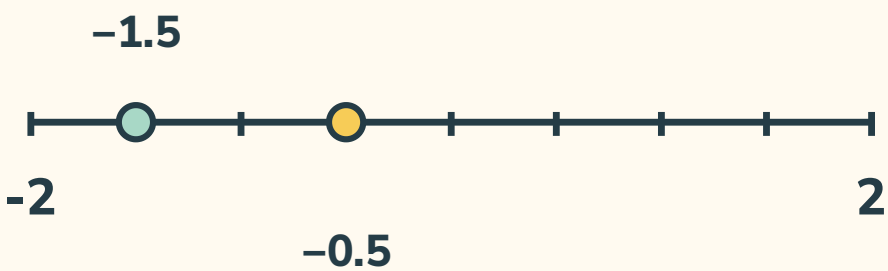
MATHS FIELD GUIDE · GRADE 9

# Density and decimal representations

OPENING PROBLEM

## Which negative number is greater?



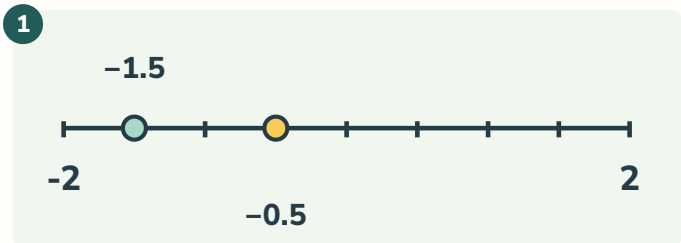

02 Locate, compare and bisect
03 Rules and worked cases
04 From the givens to the answer
05 Use the place, including the zero
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

# Locate, compare and bisect

Equal distances on a number line represent equal differences.

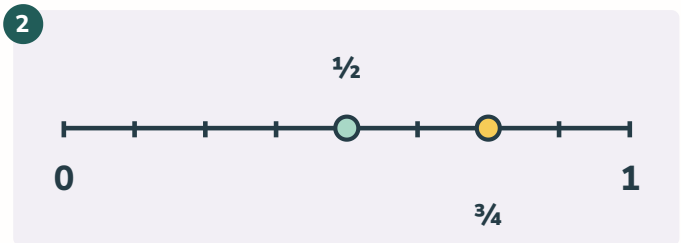
1




### Which negative number is greater?

−0.5 lies to the right. Both are negative, but −0.5 is closer to zero.

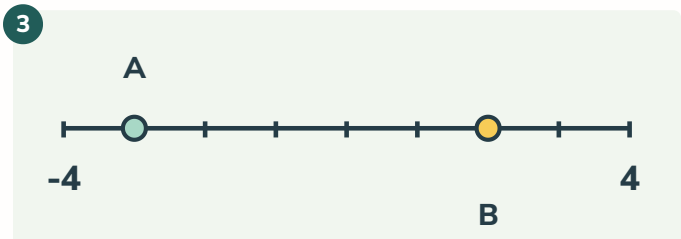
2




### Find a rational number between the two marks.

Average 1/2 and 3/4 to obtain 5/8. It lies one eighth from each endpoint.

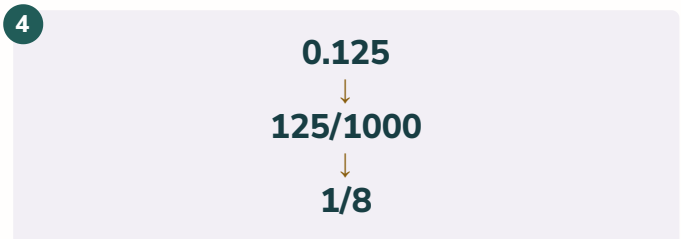
3




### How far apart are A and B?

Cross zero: three units to zero and two more to B. Distance is five.

4




### Is a terminating decimal a point on the rational line?

Yes. It equals a ratio of integers with nonzero denominator.

**Between −1 and 0**
**−1/2 is one choice.**

**Between −1 and −1/2**
**−3/4 is one choice.**

CHAPTER PROBLEM

### Find the midpoint of 2/3 and 5/6.

3/4. Add the endpoints, 2/3 and 5/6, then halve their sum to get 3/4.

MATHS REFERENCE · GRADE 9

# Density and decimal representations

## Rules, conditions and worked cases

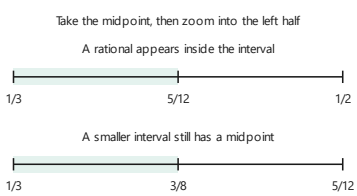
| Recognise the case | Mathematical reasoning |
|---|---|
| **Between two rationals** | Their mean lies strictly inside. Between 3/8 and 1/2 lies 7/16. |
| **When does division terminate?** | Reduce first. A denominator with only prime factors 2 and 5 terminates: 9/40 = 0.225. |
| **Convert a repeating decimal** | If x = 0.272727..., then 100x − x = 27. Thus x = 27/99 = 3/11. |

## Read the representations

### There is always room for another rational

If a < b, their average m = (a + b)/2 lies strictly between them.

When a and b are rational, adding them and dividing by 2 produces another rational. Repeat on either smaller interval.




**Between 1/3 and 1/2, the midpoint is 5/12. Smaller intervals still contain rational numbers.**

### A decimal records division

Because 1 < 6, the whole-number part of 1 ÷ 6 is 0. For the decimal digits, multiply each remainder by 10 and divide again.

A zero remainder gives a terminating decimal. A repeated remainder repeats the later digits.

Long division of 1 by 6

| Number divided | Next digit | Remainder |
|---|---|---|
| 10 | 1 | 4 |
| 40 | 6 | 4 |
| 40 | 6 | 4 again |

**1/6 = 0.1666...**

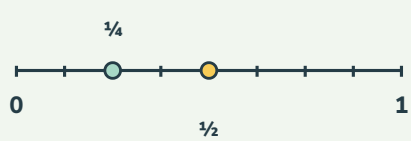
**In 1 ÷ 6, the remainder 4 returns. The digit 6 then repeats forever.**

### Which fractions terminate?

First reduce the fraction to lowest terms.

A rational decimal terminates exactly when the reduced positive denominator has only prime factors 2 and/or 5, including denominator 1.

A power of 10 contains only factors 2 and 5. This is why those denominators can be changed to 10, 100, 1000, ... .




### Find a rational point between these marks.

(1/4 + 1/2)/2 = 3/8. Its distance to either endpoint is 1/8.

WORKED EXAMPLES

# From the givens to the answer

### Are 2/5 and 3/5 consecutive rationals?

A learner sees consecutive numerators and says that no rational fits between them.

1. Use denominator 10: 2/5 = 4/10 and 3/5 = 6/10.
2. 5/10 = 1/2 lies between them.
3. Repeat the midpoint construction to produce as many new rationals as needed.

### Find three rationals between 1/4 and 1/2

1. Rename both using denominator 16: 1/4 = 4/16 and 1/2 = 8/16.
2. Choose 5/16, 6/16 and 7/16.
3. Their numerators lie strictly between 4 and 8, so all three lie inside the interval.

### Turn 0.272727... into a fraction

1. Let x = 0.272727... with the block 27 repeating forever.
2. Multiply by 100: 100x = 27.272727... .
3. Subtract x: 99x = 27.
4. Therefore x = 27/99 = 3/11.

## Check another case

### Example 1

Which rational lies strictly between 0.31 and 0.32?

0.3 · 0.315 · 0.32 · 0.325

0.315

0.310 < 0.315 < 0.320.

### Example 2

Explain why there is no smallest rational greater than 1/2.

Suppose r is rational and greater than 1/2.

The midpoint (r + 1/2)/2 is rational and lies strictly between 1/2 and r.

Thus every candidate has a smaller rational above 1/2.

**0.3 and 0.31**
↓
**0.300 and 0.310**
↓
**0.305 lies between**

### Are adjacent hundredths adjacent real numbers?

No. Thousandths and finer subdivisions still fit between them.

ILLUSTRATED PROBLEM SHEET

# Use the place, including the zero

A digit one place to the left is worth ten times as much.

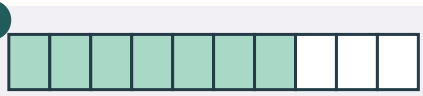
1

| Ones | Tenths | Hundredths |
|---|---|---|
| 4 | 0 | 6 |
| 4 | 6 | 0 |

### Compare 4.06 and 4.60.

The ones match. Six tenths is more than zero tenths, so 4.60 is larger.

2




### How many hundredths equal seven tenths?

Seventy. Splitting each tenth into ten parts makes 0.70.

3




### What is halfway between 0.4 and 0.5?

0.45. From 0.40 to 0.50 there are ten equal steps of 0.01. Five steps from either end reach 0.45.

4

3.08 + 0.7
↓
3.08 + 0.70
↓
3.78

### Why put the added seven under tenths?

It represents seven tenths. The eight hundredths stay unchanged.

**0.4 and 0.40**
**same point.**

**0.04 and 0.4**
**tenfold difference.**

CHAPTER PROBLEM

### Find the midpoint between −1/2 and 1/4.

-1/8. (−1/2 + 1/4)/2 = (−1/4)/2 = −1/8.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

# Check your reasoning

| Try the problem | Solution and reasoning |
|---|---|
| **01**<br>Is there a last rational number before 1? | No. Given any rational r below 1, the midpoint (r + 1)/2 is another rational strictly between them. |
| **02**<br>Which fraction has a terminating decimal?<br>A) 1/6 B) 7/20 C) 2/9 D) 5/12 | 7/20<br>$20 = 2^2 \times 5$, so 7/20 terminates. |
| **03**<br>What is 0.01 less than 2.00? | 1.99. Exchange one whole for ten tenths and a tenth for ten hundredths. |
| **04**<br>Which list contains three rationals strictly between 2/7 and 3/7?<br>A) 2/7, 5/14, 3/7 B) 9/28, 10/28, 11/28<br>C) 1/7, 2/7, 3/7 D) 3/7, 4/7, 5/7 | 9/28, 10/28, 11/28<br>The endpoints are 8/28 and 12/28. Numerators 9, 10, 11 lie inside. |
| **05**<br>Which fraction has a recurring decimal?<br>A) 3/8 B) 9/25 C) 7/30 D) 6/15 | 7/30<br>7/30 is reduced and its denominator includes a factor 3. The others reduce to denominators with only 2 and 5. |
| **06**<br>Write 0.4444... as a fraction, with 4 repeating forever. | 4/9<br>If x = 0.4444..., then 10x − x = 4, so x = 4/9. |

**7/40 = 0.175**
↓
**1/6 = 0.1666...**

## Why does one decimal terminate?

In the reduced fraction 7/40, the denominator has only 2 and 5 as prime factors. The denominator 6 also contains a 3.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

# Check your reasoning

| Try the problem | Solution and reasoning |
|---|---|
| **07**<br>Write 0.375 as a fraction. | 3/8<br>As a fraction in lowest terms the value is 3/8. Dividing numerator by denominator gives 0.375. |
| **08**<br>What happens to the decimal of 6/15 after reducing?<br>A) It recurs because 15 includes 3<br>B) It terminates because 6/15 = 2/5<br>C) It becomes irrational D) It equals 0.6 | It terminates because 6/15 = 2/5<br>Cancel 3 to obtain 2/5 = 0.4. |
| **09**<br>Prove that $a < (a + b)/2 < b$ whenever $a < b$. | From $a < b$, add a to get $2a < a + b$. Divide by 2 to get the left inequality.<br>Add b to $a < b$ to get $a + b < 2b$. Divide by 2 for the right inequality.<br>If a and b are rational, their average is rational. |
| **10**<br>Why must the decimal of a rational number terminate or eventually repeat? | For a positive integer denominator q, possible remainders are 0 through q − 1.<br>A zero remainder terminates the division.<br>If zero never occurs, a remainder must repeat, and the same later digits recur. |
| **11**<br>Write 0.636363... as a fraction, with 63 repeating. | 7/11<br>$100x - x = 63$, so $x = 63/99 = 7/11$. |

**x = 0.272727...**
↓
**100x − x = 27**
↓
**x = 27/99 = 3/11**

### What does the repeating block allow?

Shift by two decimal places so the infinite tails cancel on subtraction.

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