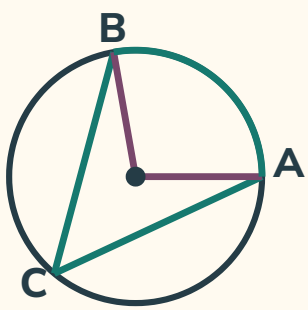
MATHS FIELD GUIDE · GRADE 9

Cyclic quadrilateral reasoning

OPENING PROBLEM

A point lies on the opposite arc. What angle does it see at the circumference?



arc 100°

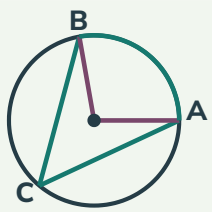
02 Identify which arc the angle sees
03 Rules and worked cases
04 From the givens to the answer
05 Compare the arguments
06 Split or specialise a quadrilateral
07 Check your reasoning
08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Identify which arc the angle sees

A central angle and an inscribed angle must intercept the same arc before using the factor of two.

1

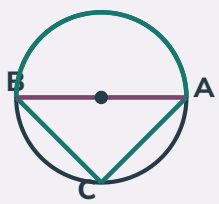


arc 100°

A point lies on the opposite arc. What angle does it see at the circumference?

Half of 100°, so 50°.

2

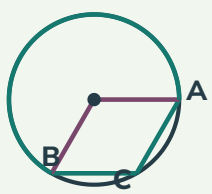


arc 180°

A triangle has this diameter as a side and its third vertex elsewhere on the circle. Find the third angle.

It intercepts a semicircle. Half of 180° is 90°.

3



arc 240°

A vertex on the minor arc sees the major arc. Find its inscribed angle.

Use 240° rather than the minor 120°. The angle is 120°.

4

Arc AB = 140°
↓
Remaining arc = 220°
↓
70° + 110° = 180°

Why are opposite angles of a cyclic quadrilateral supplementary?

Their intercepted arcs together make a full turn. Half of 360° is 180°.

Arc of 80° opposite the vertex
inscribed angle 40°.

Arc of 280° opposite the vertex
inscribed angle 140°.

CHAPTER PROBLEM

ABCD is cyclic and angle A = 103°. Find angle C in degrees.

77°. Opposite angles of this cyclic quadrilateral sum to 180°. Subtract 103° to obtain 77°.

MATHS REFERENCE · GRADE 9

Cyclic quadrilateral reasoning

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Opposite interior angles	If A = 73°, then C = 107° for a cyclic quadrilateral ABCD.
Exterior angle	Extend BC at C. The exterior angle there equals interior angle A, since both supplement C.
Converse	A convex quadrilateral with A = 64° and C = 116° is cyclic because their sum is 180°.

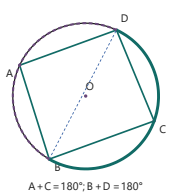
Read the representations

Cyclic means all four vertices lie on one circle

A cyclic quadrilateral has four distinct vertices on a single circle, taken in boundary order. Its sides are chords, and it is convex.

The circle is its circumcircle. The centre need not be inside the quadrilateral.

Opposite pairs are A and C, and B and D; adjacent pairs share a side.



A + C = 180°; B + D = 180°

The converse is a test for concyclicity

For a convex quadrilateral, if one pair of opposite interior angles totals 180°, the four vertices lie on a circle. This is the converse cyclic-quadrilateral theorem.

Only one opposite pair is needed: the 360° quadrilateral total then makes the other pair supplementary too.

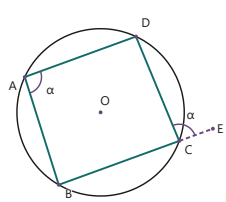
A pair of adjacent supplementary angles is insufficient. Many non-cyclic parallelograms satisfy that adjacent-angle condition.

An exterior angle equals the opposite interior angle

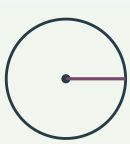
Extend BC beyond C to E. Interior angle BCD and exterior angle DCE form a straight angle.

Since A + C = 180° and C + exterior = 180°, subtract C to get exterior angle DCE = angle DAB.

Keep the extension direction explicit. The equal angle is at the opposite vertex A, not the adjacent vertex B.



B, C and E lie on one straight line



r = 5

What is the diameter?

Two radii laid end to end through the centre give 10.

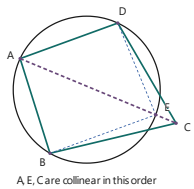
WORKED EXAMPLES

From the givens to the answer

Opposite angles halve arcs that complete the circle

1. Angle A intercepts arc BCD, which does not contain A. Opposite angle C intercepts the other arc DAB.
2. Those two arcs together total 360°. Each inscribed angle is half the measure of its intercepted arc.
3. Therefore A + C = 180°. The same argument gives B + D = 180°. Opposite angles are supplementary.

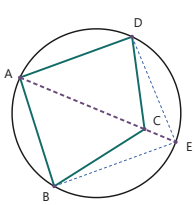
Why the converse excludes a point outside the circle



A, E, C are collinear in this order

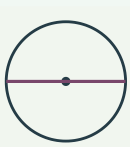
1. Draw the circle through A, B and D. Let ray AC meet it again at E. Convexity puts the crossing of diagonals AC and BD inside the circle, so E lies beyond BD. If C is outside, the order is A, E, C.
2. Both the proposed angle condition and the cyclic quadrilateral ABED give angle A + the opposite angle = 180°. Therefore they require angle BCD = angle BED.
3. At E, angle BEA is exterior to triangle BEC, so BEA > BCE. Similarly, DEA > DCE. Add these inequalities: BED > BCD.
4. That contradicts the required equality. The diagonal AC splits both compared angles into two parts, so the argument does not depend on a side meeting the circle again.

The inside case gives the reverse contradiction



A, C, E are collinear in this order

1. Use the same circle through A, B and D and extend diagonal AC to the circle at E. If C is inside, the order is A, C, E.
2. The proposed opposite-angle condition again requires BCD = BED, since ABED is cyclic.
3. Now angle BCA is exterior to triangle BCE, so BCA > BEC. Similarly, DCA > DEC. Adding gives BCD > BED, contradicting equality.
4. C can be neither outside nor inside the circle. It must lie on the circle, proving the converse for a convex quadrilateral.



d = 12

Which chord is longest?

A diameter. Its length is twice the radius, here 12.

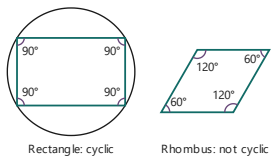
WORKED APPLICATIONS

Compare the arguments

Solve an angle expression after identifying opposite vertices

1. A convex cyclic quadrilateral has opposite angles A = 2x + 14 degrees and C = 3x – 9 degrees.
2. They total 180°, so 5x + 5 = 180 and x = 35. The angles are A = 84° and C = 96°.
3. Check that both are between 0° and 180° and that their sum is 180°. Their equality was never required.

The converse identifies rectangles, but not every parallelogram



Rectangle: cyclic Rhombus: not cyclic

1. Every rectangle has opposite angles 90° + 90° = 180°, so every rectangle is cyclic. A square is a rectangle, so it is cyclic too.
2. In any parallelogram the opposite angles are equal. If it is also cyclic, equal opposite angles must each be 90°.
3. Thus a cyclic parallelogram is a rectangle. A sloping rhombus with 60° and 120° angles fails the cyclic test.

Does any supplementary pair establish a cyclic quadrilateral?

A learner uses adjacent angles of 72° and 108° to declare a quadrilateral cyclic.

1. Adjacent angles are not the pair specified by the converse.
2. A non-rectangular parallelogram has supplementary adjacent angles but equal, non-supplementary opposite angles.
3. Label the vertices in order and test A with C or B with D.

Check another case

Example 1

Opposite cyclic angles are 4x and 5x degrees. Find x.

20

9x = 180 gives x = 20.

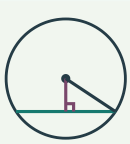
Example 2

How would you test whether four points forming a convex quadrilateral are concyclic using angles?

Label the four points in boundary order.

Establish that one pair of opposite interior angles totals 180°.

Apply the converse cyclic-quadrilateral theorem; adjacent supplementary angles alone do not suffice.



A radius is 13 and its centre is 5 from a chord. Find the chord length.

The perpendicular halves the chord. Half-length = √(169 – 25) = 12. Full length = 24.

ILLUSTRATED PROBLEM SHEET

Split or specialise a quadrilateral

Check the condition for each area rule before using it.

1

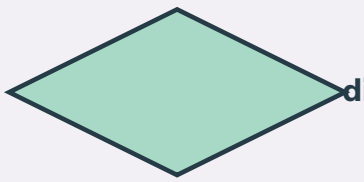


8 by 3

Check its area using a diagonal.

Each of the two right triangles has area 12. Together they give 24 square units.

2



diagonals 6, 8

Why is the area half the product of the diagonals?

They meet at right angles. The four pieces each have legs 3 and 4 and area 6. Total 24.

3

Cyclic sides 3, 4, 3, 4

↓

$s = 7$

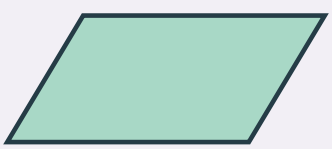
↓

$\text{Area}^2 = 4 \times 3 \times 4 \times 3$

Use the cyclic quadrilateral area formula.

$\sqrt{144} = 12$. A 3 by 4 rectangle provides a check.

4



sides 3, 4

Does the side list alone always force area 12?

No. Tilting the sides can change the height. The cyclic condition matters for the side-only formula.

A square of side 5
area 25.

A non-square rhombus of side 5
area depends on its height.

CHAPTER PROBLEM

PQRS is cyclic, in boundary order. Angle Q is 118°. Find angle S in degrees.

62°. Opposite angles of this cyclic quadrilateral sum to 180°. Subtract 118° to obtain 62°.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 In one circle, can equal chords have different minor central angles?	No. Triangles formed by the equal chords and two pairs of radii are congruent by SSS.
02 Which condition proves that a convex quadrilateral is cyclic? A) Two sides have equal lengths only B) One opposite angle pair totals 180° C) One adjacent angle pair totals 180° D) Its diagonals have equal lengths only	One opposite angle pair totals 180° This is the converse cyclic-angle theorem.
03 What extra measurement lets you find the area of a parallelogram with a known base?	The perpendicular height to that base. Multiply the two.
04 ABCD is cyclic. BC is extended beyond C to E. Angle A is 79°. Find angle DCE in degrees.	79° The exterior angle at C equals the opposite interior angle at A.
05 Opposite angles of a cyclic quadrilateral are x + 26 and 2x + 13 degrees. Find x.	47 3x + 39 = 180 gives x = 47.
06 A convex quadrilateral has opposite angles 82° and 98°. Which conclusion follows? A) It is necessarily a square B) Its sides are all equal C) Its diagonals are perpendicular D) It is cyclic	It is cyclic The opposite angles total 180°, satisfying the converse.



Which part is one quarter of the circumference?

The curved arc between the two radii. The filled sector is one quarter of the disk area.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Which parallelograms are cyclic? A) None B) Exactly the rectangles, including squares C) All parallelograms D) Only non-square rhombuses	Exactly the rectangles, including squares Opposite angles are both equal and supplementary, so each is 90°.
08 A convex quadrilateral has angles 70°, 110°, 70°, 110° in order. Is it cyclic? A) No: cyclic quadrilaterals cannot have obtuse angles B) No: neither opposite pair totals 180° C) Yes: each adjacent pair totals 180° D) Yes: every quadrilateral is cyclic	No: neither opposite pair totals 180° Opposite pairs total 140° and 220°, so the necessary cyclic condition fails.
09 Prove the opposite-angle property of a cyclic quadrilateral using arcs.	Identify the intercepted arc for each of a pair of opposite angles. These two arcs complete the circle and total 360°. Each angle is half its arc measure, so the opposite angles total 180°.
10 Explain why a non-square rhombus cannot be cyclic.	A rhombus is a parallelogram, so its opposite angles are equal. If it were cyclic, its opposite angles would also be supplementary, making every angle 90°. A rhombus with four right angles is a square, contradicting the non-square condition.
11 An exterior angle at a vertex of a cyclic quadrilateral is 134°. Find the opposite interior angle in degrees.	134° The exterior angle equals the opposite interior angle.

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