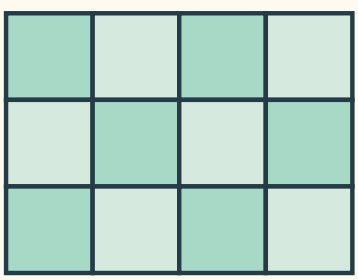
MATHS FIELD GUIDE · GRADE 9

Cube and cuboid area and volume

OPENING PROBLEM

One layer of unit cubes forms this grid. How many cubes?



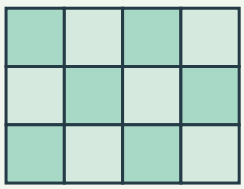
02 Build a solid from equal layers
03 Rules and worked cases
04 An open top removes one face
05 From the givens to the answer
06 Count only the exposed surfaces
07 Check your reasoning
08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Build a solid from equal layers

Volume = area of one layer × perpendicular height for a prism.

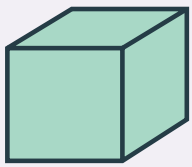
1



One layer of unit cubes forms this grid. How many cubes?

Three rows of four make 12 cubes.

2



How many 1 cm cubes fill this cube?

Nine fit in a layer. Three layers give 27 cm³.

3

Layers	Volume (cm³)
1	12
2	24
5	60

Which box has volume 60 cm³ if its base is 12 cm²?

The height is 5 cm. Divide the volume by the base area.

4

240 cm³
↓
240 mL
↓
0.24 L

How does water help compare capacity and volume?

One cubic centimetre of internal space holds one millilitre.

Base 8 cm², height 3 cm
24 cm³.

Volume 24 cm³, height 6 cm
base 4 cm².

CHAPTER PROBLEM

A cuboid is 7 cm by 4 cm by 5 cm. Find its volume in cm³.

140. Volume = 7 × 4 × 5 = 140.

MATHS REFERENCE · GRADE 9

Cube and cuboid area and volume

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Cuboid 2 × 3 × 5	Volume = 30 cubic units; surface area = 2(6 + 15 + 10) = 62 square units.
Cube with surface area 150	$6s^2 = 150$, hence s = 5 and volume = 125.
Paint a closed box	Count only exterior faces. The two cuboid volumes add. Their shared faces are hidden from the paint.

Read the representations

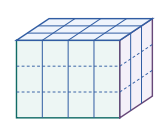
Volume counts space in three dimensions

A cuboid has rectangular faces and three perpendicular dimensions: length ℓ, width w and height h.

A unit-thick layer contains ℓw unit cubes when the dimensions are whole numbers. Stacking h layers gives volume ℓwh. Subdividing the units extends the same product to fractional dimensions.

The pictured cuboid has a 4 by 3 layer repeated three times. Volume uses cubic units; it is different from the area of its outside faces.

Volume = base area × height



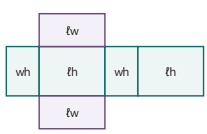
4 × 3 per layer; 3 layers

A net inventories the six faces

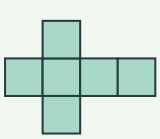
Opening a closed cuboid along edges gives six rectangles. Opposite faces have equal dimensions.

There are two ℓw faces, two ℓh faces and two wh faces. Their total area is 2(ℓw + ℓh + wh).

A net preserves each face's area. It does not preserve the shape of the solid's outline in a perspective drawing.



Two ℓw + two ℓh + two wh



A strip of four makes the sides. What do the two spare squares cover?

One closes the top and one closes the bottom.

THEOREMS AND REPRESENTATIONS

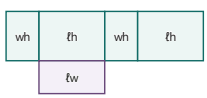
An open top removes one face

An open top removes one face

For a thin open-top cuboid, the base remains but the top face is absent. The exposed face inventory is $\ell w + 2\ell h + 2wh$.

The four walls alone have lateral area $2h(\ell + w)$. Add a base only if the task includes it.

These formulae describe one set of thin faces. Painting both sides, adding thickness or covering a rim requires a separate inventory.



Base + four walls; no top face

A cube is the equal-edge special case

Set $\ell = w = h = a$ in the cuboid formulae. The cube volume becomes a^3 and its total area becomes $6a^2$.

Each of its six faces is a square of area a^2. An open-top thin cube has five such faces, giving $5a^2$.

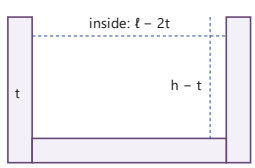
The cube formula is obtained by adding an equal-edge condition to the cuboid, rather than by introducing a different counting method.

Capacity uses the hollow space inside

An open box has outside dimensions ℓ, w, h and uniform wall and base thickness t. Opposite walls reduce the internal length and width by 2t.

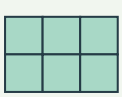
Its internal depth is $h - t$ because there is a base but no lid. Internal capacity is $(\ell - 2t)(w - 2t)(h - t)$.

These dimensions must remain positive. A closed hollow box with equally thick top and bottom would instead have internal height $h - 2t$.



outside length ℓ; outside height h

Open top: internal depth loses one base thickness



Can this two-by-three rectangle close into a cube?

No. Four squares meet at an interior corner, but only three cube faces meet at a vertex.

WORKED EXAMPLES

From the givens to the answer

Use one set of dimensions to answer two different questions

1. A closed box has length 8 cm, width 5 cm and height 3 cm. Its volume is 8 × 5 × 3 = 120 cm³.
2. Its total outside area is 2(40 + 24 + 15) = 158 cm². If the top is removed in a thin-sheet model, subtract 40 cm² to obtain 118 cm².
3. At ₹2 per cm², covering the closed box's outside costs ₹316. Multiply the area by an area price, not by a price per unit length.

Can outside volume be reported as capacity?

A learner multiplies a thick container's outside dimensions to find how much liquid it can hold.

1. The product includes the walls and base as well as the hollow space.
2. First find the internal dimensions using the stated thickness and whether a top is present.
3. For example, outside dimensions 12 × 10 × 7 cm with 1 cm walls and base give an open-box interior of 10 × 8 × 6 cm, or 480 cm³.

Check another case

Example 1

A task asks for the cost of lining a tank's inner walls and floor. Explain how to choose the dimensions and faces.

Use the internal length, width and depth for the lining.

Add the floor and four inner wall areas, excluding an open top.

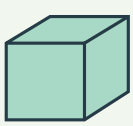
Multiply that area by the stated price per unit area, with compatible units.

Example 2

A cuboid is 9 cm by 6 cm by 4 cm. Find its volume in cm³.

216

9 × 6 × 4 = 216.



6 faces

How many edges does a cube have?

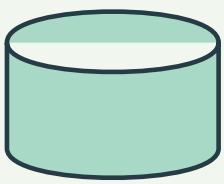
Six faces have four edges each. Each edge belongs to two faces, so 24 ÷ 2 = 12.

ILLUSTRATED PROBLEM SHEET

Count only the exposed surfaces

List the parts before adding their areas. A joined face disappears inside the finished solid.

1

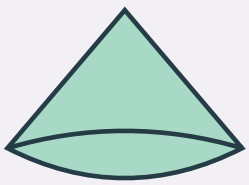


r = 2, h = 5

Which area is exposed on an open-top cylinder with a base?

Curved outside 20π plus base 4π gives 24π square units.

2



r = 3, l = 5

Find the curved area and the total area.

πrl = 15π. Add the circular base 9π for 24π square units.

3



r = 3

A solid hemisphere rests on its flat face. What is its curved area?

Half the full sphere's surface gives $2\pi r^2 = 18\pi$. Count another 9π only if the flat face is exposed.

4

Cylinder r = 2, h = 5

↓

Matching hemisphere on top

↓

20π + 8π + 4π

Find the area including the bottom.

32π square units. Count the cylinder wall, hemispherical dome and bottom. The joining disks are internal.

Remove a cylinder lid
subtract πr^2 from closed surface area.

Join two matching flat disks
subtract $2\pi r^2$ from the sum of exposed areas.

CHAPTER PROBLEM

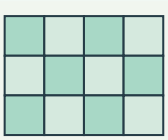
A closed cube has edge 6 cm. Find its total surface area in cm².

216. Six faces each have area 36, giving 216.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Double the three dimensions of a box. How does volume change?	Two times as many cubes fit in each of three directions. The factor is eight.
02 If every length is tripled, must the painting cost triple at a fixed price per unit area?	Exposed area scales by nine, so the cost scales by nine.
03 A closed cuboid is 10 cm by 3 cm by 4 cm. Find its total area in cm^2.	164 $2(30 + 40 + 12) = 164$.
04 A thin open-top box has base 12 cm by 5 cm and height 4 cm. Find its face area in cm^2.	196 Base 60 plus walls $2 \times 4 \times (12 + 5) = 136$ gives 196.
05 A cube has volume 343 cm^3. Find its edge length in cm.	7 $7^3 = 343$, so the positive edge length is 7.
06 An open box has outside dimensions 14 × 12 × 9 cm and uniform wall/base thickness 1 cm. Find its capacity in cm^3.	960 The inside is $12 \times 10 \times 8$, giving 960.



Cut a 4 by 3 rectangular face from card. Is this one face of a cube?

No. Cube faces are squares. It can be a face of a cuboid.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 A cuboid's outside area is 230 cm². Covering it costs ₹3 per cm². Find the total cost in rupees.	690 230 × 3 = 690.
08 Derive the total surface area of a closed cuboid from its net.	Identify the two faces with area ℓw. Identify the pairs with areas ℓh and wh. Adding all six faces gives 2(ℓw + ℓh + wh), in square units.
09 Why does an open box's internal depth lose one thickness while its width loses two?	The depth is reduced by the base thickness, with no lid at the top. The width runs between two opposite walls, each with thickness t. Thus depth is h − t and width is w − 2t, assuming uniform thickness and positive interior dimensions.
10 A cube has edge 9 cm. Find its volume in cm³.	729 $9^3 = 729$.
11 A thin open-top cube has edge 8 cm. Find its face area in cm².	320 Five square faces each have area 64, giving 320.

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