MATHS FIELD GUIDE · GRADE 9

Counting outcomes without double-counting

OPENING PROBLEM

Choose a captain then a deputy from three pupils. How many results?

First	Possible next
A	B or C
B	A or C
C	A or B

02 Count outcomes exactly once
03 Rules and worked cases
04 Or includes either event and their overlap
05 From the givens to the answer
06 Count paths through two stages
07 Check your reasoning
08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Count outcomes exactly once

The counting unit depends on whether order matters and whether an object can be used twice.

1

First	Possible next
A	B or C
B	A or C
C	A or B

Choose a captain then a deputy from three pupils. How many results?

Three choices, then two remaining choices. Six ordered outcomes.

2

AB and BA
↓
Same two-person team
↓
6 ÷ 2 = 3

What if the two roles disappear?

Each unordered pair was counted twice. There are three teams.

3

Die A	Die B
1	5
2	4
3	3
4	2
5	1

How many ordered two-die results give sum six?

Five out of 36 for independent fair dice. The double (3,3) appears once.

4

Tickets 1 to 12
↓
Multiples of 2: 6
↓
Multiples of 3: 4

How many are multiples of 2 or 3?

Subtract the two overlaps, 6 and 12. The union has 6 + 4 − 2 = 8 tickets.

Four pupils in two distinct roles
4 × 3 = 12.

Two-person teams from four
12 ÷ 2 = 6.

CHAPTER PROBLEM

Two independent fair coins are tossed. Find P(two tails).

1/4. Only TT belongs to the event among four equal ordered outcomes.

MATHS REFERENCE · GRADE 9

Counting outcomes without double-counting

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Ordered dice	(2,5) and (5,2) are distinct when dice are labelled. A sum of seven has six ordered outcomes.
Overlapping events	From 1–12, evens or multiples of 3 count 6 + 4 – 2 = 8. Subtract the shared 6 and 12 once.
Unordered selection	From A, B, C choose two different names. AB, AC, BC give three pairs; reversing a pair adds no new selection.

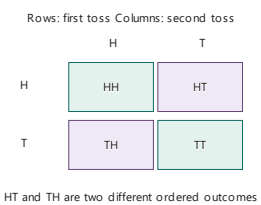
Read the representations

Record each stage in its correct position

When a coin is tossed twice, record first toss then second toss. HT and TH are different ordered outcomes even though each contains one head.

The complete space is {HH, HT, TH, TT}. With independent fair tosses, the four outcomes are equally likely.

Grouping these into zero, one or two heads gives groups of different sizes: one head contains two elementary outcomes.

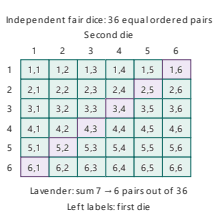


A grid checks every combination systematically

Use one die result for the row and the other die result for the column. Every cell is one ordered pair, giving 6 × 6 = 36 cells.

For independent fair dice, the 36 pairs have equal probability. The highlighted cells show the event that the sum is 7.

Different sums have different numbers of cells. Listing sums 2 through 12 does not create eleven equally likely outcomes.





Four same-sized counters differ only in colour. On a random draw, which colour is likelier?

Red has three chances out of four. Blue has one.

THEOREMS AND REPRESENTATIONS

Or includes either event and their overlap

Or includes either event and their overlap

The union A ∪ B means A or B or both. The intersection A ∩ B means both A and B.

The membership regions use uniformly selected tickets 1–12. A is even and B is a multiple of 3. Tickets 6 and 12 meet both conditions.

Separate A only, both, B only and neither so that every elementary outcome is counted once.

Tickets 1–12; A = even; B = multiple of 3

A only	A and B	B only
2	6	3
4	12	9
8		
10		

Neither: 1, 5, 7, 11

For A or B, count the shared outcomes once

Subtract the overlap when adding event counts

Adding the counts in A and B includes their shared outcomes twice. Subtract that intersection once to obtain the union count.

|A ∪ B| = |A| + |B| − |A ∩ B|. Here the vertical bars mean number of outcomes.

For mutually exclusive events the intersection is empty, so the union count is simply the sum. The blocks illustrate membership, not probability-sized areas.

A or B means A, B, or both

A only	A ∩ B	B only

|A ∪ B| = |A| + |B| − |A ∩ B|

Adding |A| and |B| counts the middle twice

Counting combinations does not itself assign chances

A menu with 3 snacks and 2 drinks has 3 × 2 = 6 snack–drink combinations when every snack can be paired with every drink.

The product counts possibilities. It gives probability 1/6 to each pair only if the selection process makes those pairs equally likely.

If some pairings are unavailable, list only the valid cells. If selections are biased, the equal-cell probability model needs revision.

Coin 1	Coin 2
H	H
H	T
T	H
T	T

Which results have exactly one head?

HT and TH. With two independent fair coins that is two of four results.

WORKED EXAMPLES

From the givens to the answer

Count a fresh union from a list

1. Choose one card uniformly from 1–20. Let A be multiples of 4 and B be multiples of 6.
2. A has 5 outcomes; B has 3. Their shared outcomes are multiples of 12: only 12 occurs in this range.
3. The union has 5 + 3 − 1 = 7 outcomes, giving P(A or B) = 7/20. The intersection probability is 1/20.

Is “one head” one outcome out of three?

A learner lists {zero heads, one head, two heads} and gives each group probability 1/3.

1. For independent fair tosses, use HH, HT, TH, TT as equal elementary outcomes.
2. One head includes HT and TH, so its probability is 2/4 = 1/2.
3. The three group labels do not have equal probabilities. Count their elementary outcomes first.

Check another case

Example 1

Describe how to check an outcome grid before using it for probability.

Confirm the row and column outcomes cover the experiment and preserve any order that matters.

Check each permitted combination appears exactly once, excluding impossible pairings.

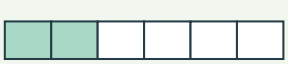
Justify equal cell chances or assign the appropriate unequal probabilities before counting an event.

Example 2

Two independent fair dice are rolled. Find P(sum 11).

1/18

(5,6) and (6,5) give 2/36 = 1/18.



2 of 6

A fair die wins on 5 or 6. What is the chance of losing?

Four other faces lose. The chance is 4/6, or 2/3.

ILLUSTRATED PROBLEM SHEET

Count paths through two stages

List ordered results or multiply probabilities along a path. State what the first draw changes.

1

First	Second
H	H
H	T
T	H
T	T

Two independent fair coins. What is P(exactly one head)?

Two of four paths, HT and TH. Probability 1/2.

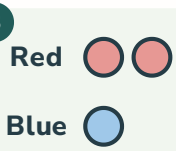
2



Two draws without replacement. What is P(two reds)?

First 2/3, then 1/2. Their product is 1/3.

3



Replace the first counter before drawing again. P(two reds)?

Now both draws have probability 2/3. Multiply to get 4/9.

4

Die 1	Die 2	Sum
1	3	4
2	2	4
3	1	4

Are there three equally likely ways to get four with two independent fair six-sided dice?

Yes. The full ordered grid has 36 cells. Three winning cells give 1/12.

Two fair independent tosses, both heads
1/4.

At least one head
1 − 1/4 = 3/4.

CHAPTER PROBLEM

Choose a ticket uniformly from 1–18. Find P(multiple of 2 or 3).

2/3. 9 even outcomes plus 6 multiples of 3 minus 3 multiples of 6 gives 12/18 = 2/3.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Is an unordered pair of die faces always as likely as any other unordered pair?	No. A non-double has two orders; a double has one.
02 Why do HT and TH count separately?	The first and second tosses occupy different positions, even when the number of heads matches.
03 Two independent fair dice are rolled. Find P(sum 8).	5/36 The pairs are (2,6), (3,5), (4,4), (5,3), (6,2).
04 Two independent fair coins are tossed. Find P(at least one tail).	3/4 HT, TH and TT qualify; only HH does not.
05 A menu offers 4 snacks and 3 drinks, with every pairing available. How many snack–drink combinations exist?	12 $4 \times 3 = 12$.

Why does the order matter here?

Choose a captain and a deputy from four pupils. There are four choices for captain and three left for deputy, giving 12 outcomes.

If the task is just to pick a two-person team, every pair was counted twice. Divide 12 by two to get six teams.



One red counter is added to a bag containing two red and two blue. Is it still fair by colour?

No. The new counts are three red and two blue.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
06 Choose uniformly from tickets 1–24. Find P(multiple of both 4 and 6).	1/12 Both means multiples of 12: 12 and 24, giving 2/24 = 1/12.
07 An equally likely space has 30 outcomes. Events A and B contain 11 and 9 outcomes, with 4 shared. Find P(A or B).	8/15 The union count is 11 + 9 − 4 = 16, so 16/30 = 8/15.
08 Why are two-dice sums 2 and 7 not equally likely in the fair independent model?	Sum 2 has only the ordered pair (1,1). Sum 7 has six ordered pairs: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1). The 36 ordered pairs are equally likely, giving probabilities 1/36 and 6/36 rather than equal sum probabilities.
09 What is the difference between A or B and A and B?	A or B is the union, containing outcomes in either event including those in both. A and B is the intersection, containing only the shared outcomes. In a union calculation, count a shared outcome once rather than once for each event label.
10 An equally likely space has 25 outcomes. Two disjoint events have 6 and 8 outcomes. Find their union probability.	14/25 No overlap exists, so (6 + 8)/25 = 14/25.

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