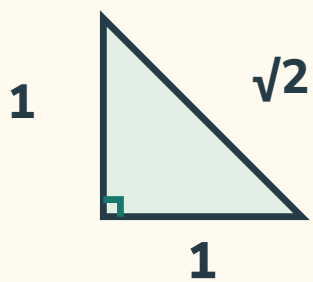
MATHS FIELD GUIDE · GRADE 9

Constructing square-root lengths

OPENING PROBLEM

From a unit segment, add another unit at right angles. What is the diagonal?



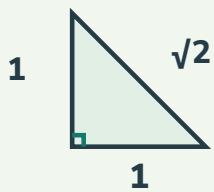
02 Construct a length from perpendicular steps
03 Rules and worked cases
04 From the givens to the answer
05 Bound a root and test the claim
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Construct a length from perpendicular steps

A new perpendicular leg gives a right triangle. Label the square of the new radius before taking the root.

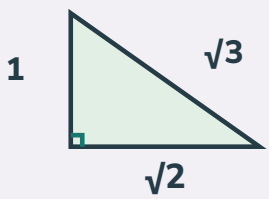
1



From a unit segment, add another unit at right angles. What is the diagonal?

Its square is 1 + 1 = 2, so its positive length is √2.

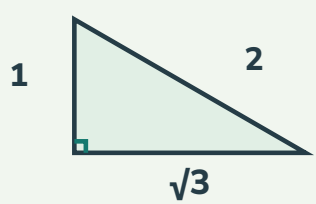
2



Build a unit leg perpendicular to the new radius. What comes next?

The squared radius becomes 2 + 1 = 3.

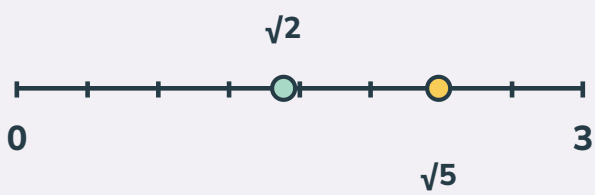
3



What happens on the next unit step?

The squared length is four. The radius is now the integer two.

4



How can a length become a point on a number line?

Set a compass to the constructed length. With centre zero, transfer it to the positive ray.

Radius squared 7, new perpendicular unit step
squared radius 8.

The length increase
√8 − √7, less than one.

CHAPTER PROBLEM

A spiral radius has squared length 8. Add a perpendicular unit segment. What is the new squared radius?

9. The new squared radius is 8 + 1 = 9.

MATHS REFERENCE · GRADE 9

Constructing square-root lengths

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Build √2	Two perpendicular unit legs produce a hypotenuse of length √2.
Extend to √3	Erect a unit perpendicular to the √2 segment at its endpoint; the new hypotenuse has square 2 + 1.
Transfer to a line	Keep the compass opening fixed at the constructed length. Arcs centred at zero meet the line at ±√3.

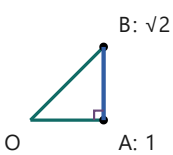
Read the representations

A unit triangle creates √2

Draw OA of length 1. At A, draw a perpendicular AB of length 1.

Pythagoras gives $OB^2 = 1^2 + 1^2 = 2$. Therefore $OB = \sqrt{2}$.

The constructed length is exact. A decimal measurement is only approximate.



Each blue outer segment has length 1. Teal radii are exact root lengths.

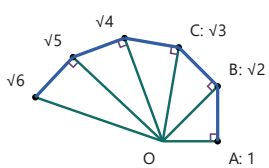
Each new unit segment is perpendicular to the current radial segment.

One more unit creates the next root

At B, draw BC of length 1 perpendicular to OB, extending the spiral outward.

$OC^2 = OB^2 + BC^2 = 2 + 1 = 3$. Therefore $OC = \sqrt{3}$.

Continuing adds 1 to the squared radial length at every step.



Each blue outer segment has length 1. Teal radii are exact root lengths.

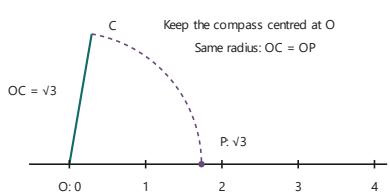
The radii progress from √1 through √6. They do not increase by a fixed length.

Transfer the radius without measuring it

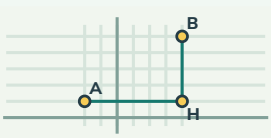
Set the compass opening to OC, the constructed length √3. Place its centre at O, which is zero on the number line.

Swing an arc from C to meet the positive number line at P. The distance OP equals OC.

For a negative coordinate, transfer the same distance to the left.



The arc preserves the radius: OC = OP = √3. The point P has coordinate √3.



A (-2, 1)
H (4, 1)
B (4, 5)

Find the direct distance from A(−2, 1) to B(4, 5).

The legs are 6 and 4. AB = √52 = 2√13. The broken route via H is 10 units.

WORKED EXAMPLES

From the givens to the answer

Does each new radius grow by 1?

A learner writes the radii as 1, 2, 3, 4 because each new outer segment is 1.

1. The added unit segment is perpendicular, not along the radius.
2. Pythagoras adds 1 to the square: $r_{new}^2 = r_{old}^2 + 1$.
3. The radii are 1, √2, √3, 2, √5, … .

Construct √5 from a length 2

1. Start with OA = 2 on a baseline.
2. At A, construct a perpendicular AB = 1.
3. $OB^2 = 2^2 + 1^2 = 5$, so OB = √5.
4. Transfer OB with a compass centred at O onto the number line.

Locate √7 relative to whole numbers

1. The spiral can reach a radial length √7 by adding perpendicular unit segments.
2. Because $2^2 < 7 < 3^2$, the transferred point lies between 2 and 3.
3. Keep √7 as the exact label. A ruler reading near 2.65 is approximate.

Check another case

Example 1

Which label preserves the exact constructed length?

1.73 exactly · √3 · 1.732 exactly · √3 = 1.7

√3

√3 is exact; finite decimal approximations need ≈.

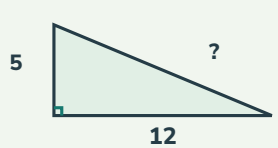
Example 2

Justify the step from radius √n to √(n + 1).

Construct an outer unit segment perpendicular to the current radius.

The new right triangle has leg squares n and 1.

Its hypotenuse has square n + 1, so its length is √(n + 1).



A right triangle has legs 5 and 12. How long is its hypotenuse?

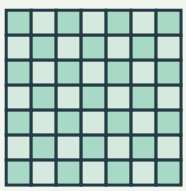
$5^2 + 12^2 = 169$, so the longest side is 13.

ILLUSTRATED PROBLEM SHEET

Bound a root and test the claim

Squaring a nonnegative number preserves order. The radical sign takes the nonnegative root.

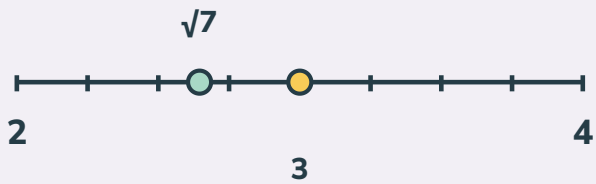
1



Which square has area 49 square units?

Seven rows of seven. Its side length is $\sqrt{49} = 7$.

2



Which integers trap √7?

$2^2 < 7 < 3^2$. Hence $2 < \sqrt{7} < 3$.

3

$$3.3^2 = 10.89$$
$$\downarrow$$
$$11$$
$$\downarrow$$
$$3.4^2 = 11.56$$

Which tenths trap √11?

$3.3 < \sqrt{11} < 3.4$. Compare squares before rounding the root.

4

$$x^2 = 25$$
$$\downarrow$$
$$x = 5 \text{ or } x = -5$$
$$\downarrow$$
$$\sqrt{25} = 5$$

Why are there two equation solutions but one square root symbol value?

Both signs square to 25. The radical denotes the nonnegative one.

$\sqrt{81}$
positive 9.

$x^2 = 81$
two real solutions, 9 and −9.

CHAPTER PROBLEM

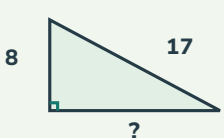
A right triangle has perpendicular legs 1 and 3. What is the square of its hypotenuse?

10. $1^2 + 3^2 = 10$. The length itself is $\sqrt{10}$.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Could the added unit segment point in an arbitrary direction and give the same calculation?	No. The perpendicular condition is what allows addition of squares.
02 Which tool transfers an exact constructed radius onto a number line? A) A calculator display B) A compass centred at the origin C) A guessed decimal label D) A change of scale without recalculation	A compass centred at the origin A compass carries the same distance without replacing it by a rounded measurement.
03 Is $\sqrt{(9 + 16)}$ equal to $\sqrt{9} + \sqrt{16}$?	No. The first is 5. The second is 3 + 4 = 7.
04 Which condition creates $\sqrt{6}$ from a radius $\sqrt{5}$? A) Add a collinear unit segment B) Add a unit segment perpendicular to the $\sqrt{5}$ radius C) Add a segment of length $\sqrt{5}$ D) Double the radius	Add a unit segment perpendicular to the $\sqrt{5}$ radius Pythagoras gives new radius squared 5 + 1 = 6.
05 A unit-leg spiral reaches radius $\sqrt{9}$. Which exact length is this? A) 3 B) 4.5 C) 9 D) 81	3 The non-negative square root of 9 is 3.
06 Starting with radius 1, how many perpendicular unit segments are added to obtain radius $\sqrt{5}$?	4 Each added segment increases the squared radius by 1: 1 to 2 to 3 to 4 to 5.



The hypotenuse is 17 and one leg is 8. Recover the other leg.

Subtract squares: 289 − 64 = 225. The missing positive length is 15.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Where does the positive transferred point √12 lie? A) Between 2 and 3 B) Between 3 and 4 C) At 6 D) Between 4 and 5	Between 3 and 4 9 < 12 < 16, so 3 < √12 < 4.
08 If a new radius is √14, what was the square of the preceding radius?	13 The new square is the previous square plus 1. So the previous square is 14 − 1.
09 Why is a perpendicular construction essential in the root spiral?	The right angle allows Pythagoras. With a unit new leg, it gives r_new² = r_old² + 1. Without the right angle that equality does not follow.
10 Describe how to place −√6 on a number line after constructing √6.	Set the compass opening to the constructed √6 radius. Centre the compass at zero. Transfer the distance to the negative side, giving the point −√6.
11 How many perpendicular unit additions take radius 1 to radius √8?	7 The squared radius starts at 1 and needs seven increases of 1 to reach 8.

$$7^2 + 24^2$$
$$\downarrow$$
$$49 + 576 = 625$$
$$\downarrow$$
$$25^2$$

Can sides 7, 24 and 25 make a right triangle?

Yes. The converse places the right angle opposite the longest side, 25.

BANANALEARN LEARNING LIBRARY

Where will your maths take you next?

GRADE 9 MATHS

Choose your next problem.

Explore more illustrated guides and practise a related topic.

Browse Grade 9 guides →

Keep exploring

GRADE 9 · TOPIC GUIDE

Using Pythagoras and its converse

Open guide →

GRADE 9 · TOPIC GUIDE

Proving square roots irrational

Open guide →

Visit bananalearn.com →