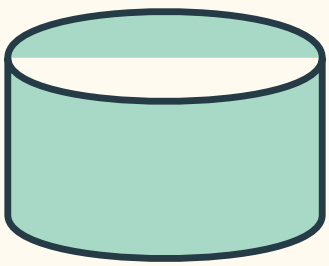
MATHS FIELD GUIDE · GRADE 9

Cone measures

OPENING PROBLEM

Find the dimensions of its unrolled wall.



r = 3, h = 5

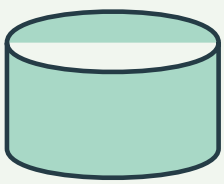
02 Unroll a curved surface
03 Rules and worked cases
04 The curved surface unfolds into a sector
05 From the givens to the answer
06 Match the solid to the formula
07 Check your reasoning
08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Unroll a curved surface

A cylinder opens into a rectangle. A cone opens into a sector whose arc is the base circumference.

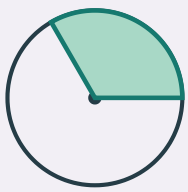
1



Find the dimensions of its unrolled wall.

Height 5 and width 6π. The curved area is 30π square units.

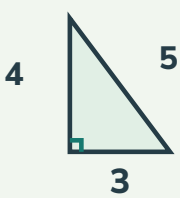
2



A cone has base radius 3 and slant height 9. Why is the sector angle 120°?

A full radius-9 circle has circumference 18π. The rim needs 6π, or one third of a turn.

3



A cone has radius 3 and vertical height 4. Which measure is the sector radius?

The slant height 5. It runs along the surface from tip to rim.

4

Curved area = π × 3 × 5

↓

Base area = 9π

↓

Total = 24π

What changes if the cone is open at the base?

Only the 15π curved surface remains.

Sector radius 6 and angle 180°
arc length 6π.

That arc becomes a cone rim
base radius 3.

CHAPTER PROBLEM

A right cone has radius 9 cm and height 12 cm. Find its slant height in cm.

15. The radius and perpendicular height are the legs. The slant height is $\sqrt{(9^2 + 12^2)} = \sqrt{225}$.

MATHS REFERENCE · GRADE 9

Cone measures

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Slant versus height	For right cone r = 5 and h = 12, slant length l = √169 = 13.
Curved surface	πrl = 65π square units. Add base area 25π for total 90π.
Volume	Use perpendicular height: πr²h/3 = 100π cubic units. Slant length does not enter.

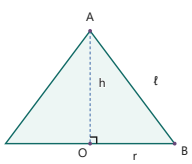
Read the representations

A right cone has two different heights

A right circular cone has its apex directly above the centre O of its circular base. The vertical height h meets the base at a right angle.

The slant height ℓ runs along the curved surface from the apex to the base rim. In the axial half-section, h and r are the perpendicular legs.

Pythagoras gives $\ell^2 = h^2 + r^2$. For a non-degenerate right cone, ℓ is greater than both h and r.



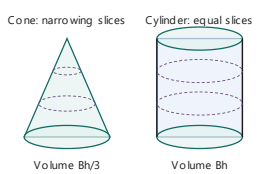
$\ell^2 = h^2 + r^2$; ℓ is the slant height

A cone narrows while a cylinder keeps equal layers

A cone and cylinder can have the same base area B and vertical height h, yet different volumes. The cone's parallel slices become smaller towards its apex.

A matched-container investigation fills the cylinder with three cone measures. The exact mathematical relationship is Vcone = Bh/3 = πr²h/3.

Like a pyramid, a cone has one third the volume of a prism or cylinder with the same base area and height. At finer and finer scales, a cone is approached by pyramids with increasingly many base sides. The height in this volume formula is h, not ℓ.



Same base B and height h; volumes in ratio 1 : 3



r = 2, h = 5

Which area is exposed on an open-top cylinder with a base?

Curved outside 20π plus base 4π gives 24π square units.

THEOREMS AND REPRESENTATIONS

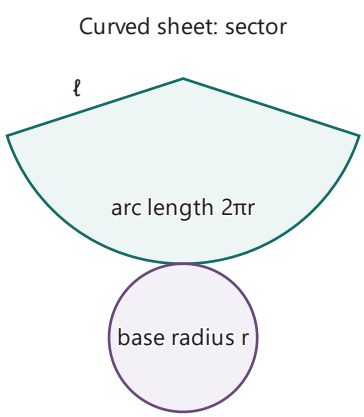
The curved surface unfolds into a sector

The curved surface unfolds into a sector

The cone's slant height becomes the sector radius ℓ. The outer arc wraps around the circular base, so its length is $2\pi r$.

A full circle of radius ℓ has circumference $2\pi\ell$. The net sector therefore occupies fraction r/ℓ of that circle.

Its area is $(r/\ell) \times \pi\ell^2 = \pi r\ell$. Thus cone curved area is $\pi r\ell$, and its closed total area adds the base disk πr^2.



Sector radius ℓ; angle fraction r/ℓ

A net must fit its base circumference

For a sector net of radius ℓ and angle $\theta°$, the fraction is $\theta/360 = r/\ell$. Hence $\theta = 360r/\ell$.

If r and ℓ are supplied instead of h, use $h = \sqrt{(\ell^2 - r^2)}$. The condition $\ell > r$ is necessary for a positive vertical height.

A sector with the wrong arc length cannot close into the requested base circle without stretching or leaving a gap.

Design a paper cone from a sector

A cone has radius 3 cm and slant height 9 cm. Its net is a sector of radius 9 cm whose arc must become the rim, length 6π cm.

A full circle of radius 9 has circumference 18π. Use one third of it, a 120° sector. Its area is 27π cm², matching πrl.



r = 3, l = 5

Find the curved area and the total area.

$\pi rl = 15\pi$. Add the circular base 9π for 24π square units.

WORKED EXAMPLES

From the givens to the answer

Find slant height before calculating curved area

1. A right cone has radius 5 cm and vertical height 12 cm. Its slant height is $\sqrt{(25 + 144)} = 13$ cm.
2. Its volume is $(1/3) \times \pi \times 25 \times 12 = 100\pi$ cm³. Its curved area is $\pi \times 5 \times 13 = 65\pi$ cm².
3. If the base is exposed too, its area 25π gives total area 90π cm². An open conical sheet would use the curved area alone.

Can slant height replace vertical height in the volume formula?

A learner uses $\pi r^2 \ell/3$ because ℓ is the visible side measurement.

1. The volume compares slices along the perpendicular height from base to apex.
2. Use Pythagoras to recover h from the given slant height and radius when necessary.
3. Use h for volume and ℓ for curved area; they answer different geometric questions.

Check another case

Example 1

A cone has radius 6 cm and vertical height 7 cm. Its volume is $k\pi$ cm³. Find k.

84

$36 \times 7/3 = 84$.

Example 2

A cone has radius 9 cm and vertical height 5 cm. Its volume is $k\pi$ cm³. Find k.

135

$81 \times 5/3 = 135$.



A solid hemisphere rests on its flat face. What is its curved area?

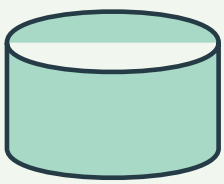
Half the full sphere's surface gives $2\pi r^2 = 18\pi$. Count another 9π only if the flat face is exposed.

ILLUSTRATED PROBLEM SHEET

Match the solid to the formula

Volume uses perpendicular height. A slant length is useful for a curved surface or a triangular face.

1

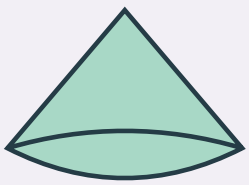


r = 3, h = 8

Find the volume of the cylinder.

Base area 9π times height 8 gives 72π cubic units.

2

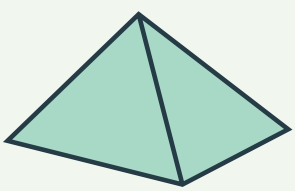


r = 3, h = 8

How much of the matching cylinder does this cone hold?

One third, or 24π cubic units. The base and perpendicular height must match.

3



B = 18, h = 5

Find the volume without using a slant length.

Bh/3 = 30 cubic units. B is the area of the base.

4



r = 3

Find the volume of a hemisphere of this radius.

Half of (4/3)π × 27 is 18π cubic units.

Prism volume V and same base and height pyramid
V/3.

A cylinder of radius r, height 2r
volume $2\pi r^3$.

CHAPTER PROBLEM

A cone has radius 3 cm and vertical height 10 cm. Its volume is kπ cm^3. Find k.

30. Take one third of the matching cylinder: $\pi \times 3^2 \times 10 \div 3 = 30\pi$.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Can a sector of radius 4 roll into a cone with base radius 5?	No. Even a full radius-4 circle has only 8π of arc. The requested rim needs 10π.
02 Two cones have equal heights but one has twice the radius. How do volumes compare?	Base area grows fourfold, so volume grows fourfold as well.
03 A right cone has radius 7 cm and vertical height 24 cm. Find its slant height in cm.	25 The radius and perpendicular height are the legs. The slant height is $\sqrt{(7^2 + 24^2)} = \sqrt{625}$.
04 A right cone has radius 8 cm and slant height 17 cm. Its curved area is kπ cm². Find k.	136 The curved area is π times radius times slant height. Its coefficient is $8 \times 17 = 136$.
05 A closed cone has radius 4 cm and slant height 9 cm. Its total area is kπ cm². Find k.	52 Curved coefficient 36 plus base coefficient 16 gives 52.
06 A right cone has radius 12 cm and slant height 13 cm. Find its vertical height in cm.	5 The slant height is the hypotenuse. Subtract 12^2 from 13^2 and take the positive square root.

Cylinder r = 2, h = 5
↓
Matching hemisphere on top
↓
20π + 8π + 4π

Find the area including the bottom.

32π square units. Count the cylinder wall, hemispherical dome and bottom. The joining disks are internal.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 A cone has base radius 6 cm and slant height 10 cm. Find the angle of its sector net in degrees.	216° $\theta = 360 \times 6/10 = 216°$.
08 Derive the cone curved-area formula using its sector net.	The net sector radius is ℓ and its arc length is $2\pi r$. Its fraction of a full radius-ℓ circle is r/ℓ. Multiply $\pi\ell^2$ by that fraction to obtain $\pi r\ell$.
09 Why can a right cone not have base radius 10 cm and slant height 8 cm?	The slant would be the hypotenuse of the radius-height right triangle. It must be longer than the radius for a positive vertical height. The proposed values give $h^2 = 64 - 100 < 0$, so they are impossible for this right cone.
10 A conical sheet has radius 11 cm and slant height 20 cm. Its curved area is $k\pi$ cm². Find k.	220 $r\ell = 11 \times 20 = 220$.
11 Explain how you would solve a cone problem given radius and slant height.	Check that $\ell > r$ for a non-degenerate right cone. Recover h by Pythagoras if volume is required. Use $\pi r\ell$ for curved area and add πr^2 only if the base is part of the exposed surface.

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