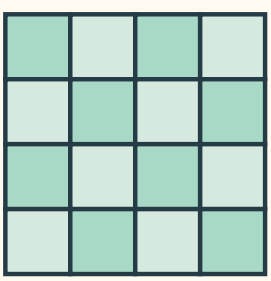
MATHS FIELD GUIDE · GRADE 9

Classical square construction and area transformation

OPENING PROBLEM

A 4 by 4 square is cut along a diagonal. What is the area of either piece?



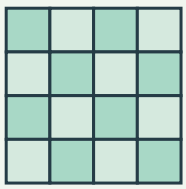
02 Transfer an area into a square
03 Rules and worked cases
04 Squaring a rectangle means preserving its area
05 From the givens to the answer
06 Split or specialise a quadrilateral
07 Check your reasoning
08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Transfer an area into a square

For a square with side s, its area is s^2 and its diagonal is $s\sqrt{2}$.

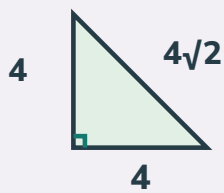
1



A 4 by 4 square is cut along a diagonal. What is the area of either piece?

Eight square units. The two congruent right triangles share the square equally.

2



Use the diagonal of that square as the side of a new square. Find the new area.

$(4\sqrt{2})^2 = 32$. The new area is twice the original 16.

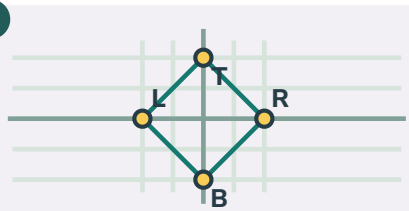
3

Rectangle 9 by 16
↓
Area = 144
↓
Square side = √144

Which square matches the rectangle's area?

The square of side 12. Matching areas need not have matching perimeters.

4



L (-2, 0)
T (0, 2)
R (2, 0)
B (0, -2)

How do you know these four endpoints form a square?

Adjacent displacements have equal length $\sqrt{8}$ and meet at right angles. The diagonals share a midpoint and length 4.

Square diagonal 6
area 18.

Square side 6
area 36.

CHAPTER PROBLEM

A required square has side 10 cm. What radius in cm is used for each of the four corner-locating circles?

5. The cord construction uses half the required side as the radius.

MATHS REFERENCE · GRADE 9

Classical square construction and area transformation

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Construct a square on AB	Erect perpendiculars at A and B on the same side. Mark off AB on both and join the new endpoints.
Check area	A square of side √13 has area 13. A 2-by-3 right triangle gives hypotenuse √13 to transfer by compass.
Double the square's area	Its diagonal has squared length $2s^2$. Use that diagonal as the side of a new square.

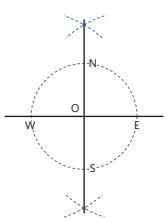
Read the representations

A cord can preserve a length while its direction changes

Baudhāyana's Śulbasūtra describes square constructions with cords and pegs. We represent the cord by compass radii and use modern labels to explain the geometry.

Let the required side be s. Mark E and W with EW = s, and its midpoint O. Draw the circle with centre O and radius s/2.

Equal arcs of radius s from E and W locate the perpendicular bisector. Its intersections N and S with the first circle give four axis points, each s/2 from O.



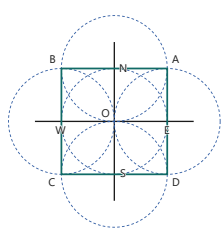
The outer arc intersections locate the north–south line. E and W lie on the east–west line.

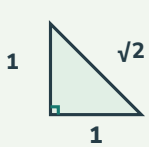
The outer intersections locate the square's corners

Draw four circles of radius s/2, one centred at each of E, N, W and S. Adjacent circles meet at O and at one outer point.

Choose the four outer points A, B, C and D, then join them in order. O is not a corner of the required square.

Each corner is s/2 from both adjoining axis points. The coordinate check on the next slide explains why the completed sides are s and the angles are right.





From a unit segment, add another unit at right angles. What is the diagonal?

Its square is 1 + 1 = 2, so its positive length is √2.

THEOREMS AND REPRESENTATIONS

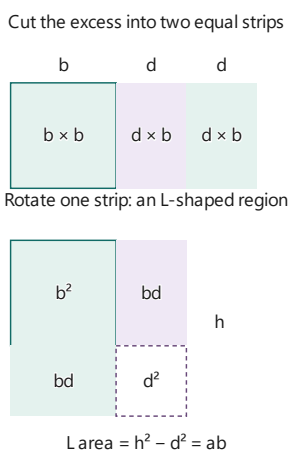
Squaring a rectangle means preserving its area

Squaring a rectangle means preserving its area

For rectangle sides a ≥ b > 0, mark off a b-by-b square. Divide the remaining length a − b into two parts of length d = (a − b)/2.

Keep one b-by-d strip beside the square and rotate the other below it. The pieces make an L with no overlap, fitting inside a square of side h = b + d = (a + b)/2.

The missing corner is a d-by-d square. Thus the original area ab equals $h^2 - d^2$. This is the area rearrangement behind the classical transformation.

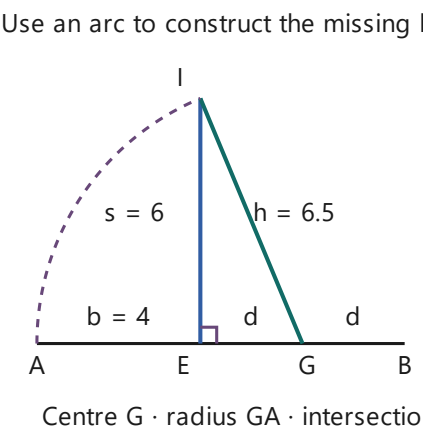


Construct a side whose square is that area

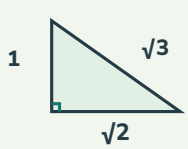
For a > b > 0, place E between A and B on the same ray, with AB = a and AE = b. Bisect EB at G: EG = GB = d and AG = h.

Erect a perpendicular to AB at E. Draw the arc with centre G and radius GA, meeting that perpendicular above the line at I.

GI = h and GE = d. The right triangle gives $EI^2 = h^2 - d^2 = ab$. Construct a square on the positive length EI to preserve the rectangle's area.



Example shown: a = 9, b = 4. The arc sets h = 6.5; the perpendicular side is 6.



Build a unit leg perpendicular to the new radius. What comes next?

The squared radius becomes 2 + 1 = 3.

WORKED EXAMPLES

From the givens to the answer

Justify the cord construction with coordinates

1. Write r = s/2. Use O = (0,0), E = (r,0), N = (0,r), W = (−r,0), S = (0,−r).
2. The outer intersection of the E and N circles is (r,r): its distance from each centre is r. The other corners are (−r,r), (−r,−r), (r,−r).
3. Each side changes just one coordinate by 2r = s. Adjacent sides are horizontal and vertical, so all four angles are 90°. The figure is a square of the required side.

Explain why the rectangle construction is exact

1. The rearrangement gives $h^2 - d^2 = ab$. Algebra confirms it: $[(a + b)^2 - (a - b)^2]/4 = 4ab/4 = ab$.
2. Pythagoras in the constructed right triangle gives $s^2 + d^2 = h^2$, so $s^2 = ab$. A square of side s has exactly the required area.
3. If a = b, d = 0 and s = a; the original shape is already a square. For a > b, the positive side is $\sqrt{(ab)}$, not generally (a + b)/2.

Does averaging the sides preserve area?

A learner replaces a 9-by-1 rectangle by a square of side (9 + 1)/2 = 5.

1. The rectangle's area is 9, while a side-5 square has area 25.
2. The equal-area side is $\sqrt{9} = 3$. Averaging the sides gives h, which is the constructed triangle's hypotenuse.
3. Use $s^2 = h^2 - d^2$, with d = 4: $s^2 = 25 - 16 = 9$.

Transform a different rectangle

1. For a = 18 and b = 8, h = 13 and d = 5.
2. Construct a right triangle with hypotenuse 13 and one leg 5. The other leg has square 169 − 25 = 144, hence length 12.
3. A square of side 12 has area 144, matching 18 × 8. The lengths are in the same unit, and the areas in square units.

Check another case

Example 1

If a = b > 0 in the rectangle transformation, then...

d = 0 and s = a · s = 0 · The area becomes $2a^2$ · The construction has no positive side

d = 0 and s = a

The rectangle is already a square; h = a and d = 0.

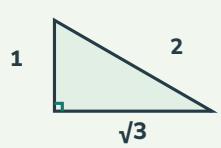
Example 2

Explain why the arc construction produces a square with area ab.

The arc gives GI = AG = h, while GE = d and EI is perpendicular to GE.

Pythagoras gives $EI^2 = h^2 - d^2$.

The difference-of-squares identity gives $h^2 - d^2 = ab$, so the square on EI has the original rectangle's area.



What happens on the next unit step?

The squared length is four. The radius is now the integer two.

ILLUSTRATED PROBLEM SHEET

Split or specialise a quadrilateral

Check the condition for each area rule before using it.

1

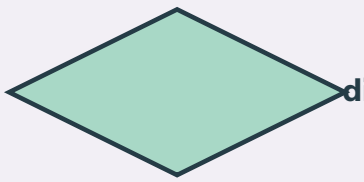


8 by 3

Check its area using a diagonal.

Each of the two right triangles has area 12. Together they give 24 square units.

2



diagonals 6, 8

Why is the area half the product of the diagonals?

They meet at right angles. The four pieces each have legs 3 and 4 and area 6. Total 24.

3

Cyclic sides 3, 4, 3, 4

↓

$s = 7$

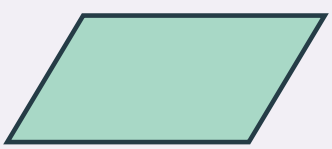
↓

$\text{Area}^2 = 4 \times 3 \times 4 \times 3$

Use the cyclic quadrilateral area formula.

$\sqrt{144} = 12$. A 3 by 4 rectangle provides a check.

4



sides 3, 4

Does the side list alone always force area 12?

No. Tilting the sides can change the height. The cyclic condition matters for the side-only formula.

A square of side 5
area 25.

A non-square rhombus of side 5
area depends on its height.

CHAPTER PROBLEM

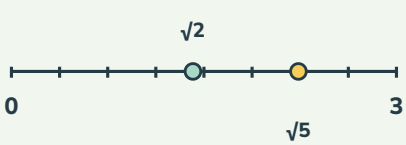
A rectangle has sides 16 and 9. Find the side of an equal-area square.

12. The area is 144 and its positive square root is 12.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Two squares have areas 9 and 16. Which square combines their areas?	A square of side 5 has area 25. Adding their sides instead would give a square of side 7.
02 What extra measurement lets you find the area of a parallelogram with a known base?	The perpendicular height to that base. Multiply the two.
03 In the cord construction, r = 3.5 cm. Find the completed square's side in cm.	7 Opposite corner coordinates differ by 2r = 7.
04 Which intersections form the constructed square? A) O and any three axis points B) The four outer intersections of adjacent equal-radius circles C) The four circle centres E, N, W and S D) Any four points on the first circle	The four outer intersections of adjacent equal-radius circles The outer intersections have coordinates (±r, ±r) and sides 2r.
05 For rectangle sides a = 15 and b = 3, find d = (a − b)/2.	6 (15 − 3)/2 = 6.
06 For rectangle sides a = 11 and b = 5, find h = (a + b)/2.	8 (11 + 5)/2 = 8.



How can a length become a point on a number line?

Set a compass to the constructed length. With centre zero, transfer it to the positive ray.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 A right triangle has hypotenuse h = 10 and one leg d = 6. Find the other positive leg s.	8 $s^2 = 100 - 36 = 64$, so s = 8.
08 What is the exact equal-area square side for a 7-by-3 rectangle? A) 5 B) 21 C) √21 D) √10	√21 $s^2 = 7 \times 3 = 21$.
09 Why does cutting and moving the two strips preserve the original area?	Cutting partitions the rectangle into one square and two strips. Moving and rotating a piece preserves its area. The rearranged pieces do not overlap and leave only the identified d-by-d corner missing from the h-by-h square.
10 Explain why a historical construction should be checked rather than accepted only because it is old.	Historical attribution tells us where a method is described. A geometric argument must still show that the construction meets its requirements. Here equal radii, perpendicular directions and exact area equalities provide that justification.
11 Find the equal-area square side for a rectangle with sides 25 cm and 4 cm.	10 Its area is 100 cm^2, giving a positive side of 10 cm.

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