MATHS FIELD GUIDE · GRADE 9

Classical probability and complements

OPENING PROBLEM

A fair die wins on 5 or 6. What is the chance of losing?



win 2/6

02 Count the event that is easier
03 Rules and worked cases
04 Count outcomes when each has the same chance
05 From the givens to the answer
06 Count outcomes exactly once
07 Check your reasoning
08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Count the event that is easier

An event and its complement share no outcomes and together exhaust the sample space.

1



A fair die wins on 5 or 6. What is the chance of losing?

Four of six faces lose. Equivalently 1 − 2/6 = 2/3.

2

Two independent fair tosses
↓
P(no heads) = ¼
↓
P(at least one head) = ¾

Why use the complement?

Only TT has no heads. This avoids separately counting HT, TH and HH.

3

P(rain) = 0.32
↓
P(no rain) = 1 − 0.32

Must “no rain” mean sunny?

P(no rain) = 0.68. A cloudy dry day also belongs to the complement.

4

Ticket	1 through 10
Event	greater than 7
Complement	at most 7

What happens to the boundary ticket 7?

It belongs to the complement. For a uniform draw the two probabilities are 3/10 and 7/10.

At least one success
complement is zero successes.

Exactly one success
the complement can include zero or several successes.

CHAPTER PROBLEM

A ticket is selected uniformly from numbers 1–10. Find the probability of selecting a number greater than 7. Enter a fraction.

3/10. The outcomes 8, 9, 10 are three of ten equally likely tickets.

MATHS REFERENCE · GRADE 9

Classical probability and complements

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Equally likely outcomes	On a fair die, numbers above four are 5 and 6, giving 2/6 = 1/3.
Complement	At least one head in three independent fair flips has chance $1 - (1/2)^3 = 7/8$.
Boundary check	The probabilities of all mutually exclusive, exhaustive outcomes sum to one.

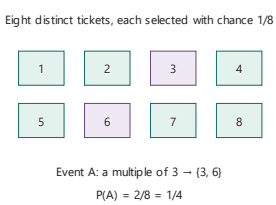
Read the representations

A random experiment has a set of possible outcomes

An experiment has a stated procedure, such as mixing tickets and selecting one without looking. An outcome is the recorded result of one trial.

The sample space S lists every possible elementary outcome once. An event A is a collection of outcomes meeting a condition.

Here S = {1, 2, 3, 4, 5, 6, 7, 8}. Selecting a multiple of 3 is the event A = {3, 6}. One selection produces one outcome, which may or may not belong to A.

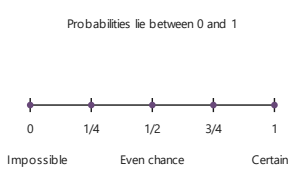


Probability measures how likely an event is

Probability is a number from 0 to 1. An impossible event has probability 0 and a certain event has probability 1 in the stated model.

A probability of 1/2 represents an even chance. Values nearer 1 represent more likely events. Fractions, decimals and percentages can express the same probability.

The model describes uncertainty before a trial. An event with probability 3/4 can still fail on the next trial.





Four same-sized counters differ only in colour. On a random draw, which colour is likelier?

Red has three chances out of four. Blue has one.

THEOREMS AND REPRESENTATIONS

Count outcomes when each has the same chance

Count outcomes when each has the same chance

For a finite equally likely sample space, P(A) = number of outcomes in A ÷ number of outcomes in S.

State why the elementary outcomes are equally likely: a fair die has equal face chances, and a uniform ticket selection gives each distinct ticket the same chance.

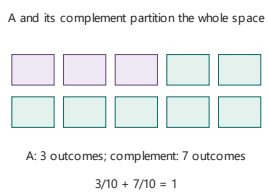
If the process favours some outcomes, a simple count ratio may fail. The word random alone does not guarantee that the listed categories have equal chances.

The complement contains every outcome outside the event

The complement of A, written not A or A^c, contains all outcomes in S that are not in A.

A and its complement have no shared outcomes, and together they exhaust the sample space. Their probabilities therefore add to 1.

Use P(not A) = 1 − P(A). The partition below shows this with ten equally likely outcomes.

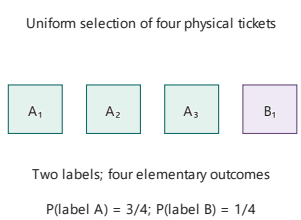


Two labels can have different probabilities

The four physical tickets below are equally likely to be selected. Three carry label A and one carries label B.

The sample space of physical tickets is {A_1, A_2, A_3, B_1}. Grouping them by label gives probabilities 3/4 and 1/4.

Listing only the two labels {A, B} does not justify giving each a probability of 1/2. Check the elementary outcomes behind a category.



Coin 1	Coin 2
H	H
H	T
T	H
T	T

Which results have exactly one head?

HT and TH. With two independent fair coins that is two of four results.

WORKED EXAMPLES

From the givens to the answer

Use the easier side of an event

1. Six separate cards carry the letters P, L, A, N, E, T. Select one card uniformly. Find the probability of selecting a consonant.
2. The vowels are A and E: two cards. The other four cards carry consonants. P(vowel) = 2/6 = 1/3.
3. P(consonant) = 1 − 1/3 = 2/3. Listing the four consonants gives the same answer directly.

Does having two possible results make both equally likely?

A learner assigns a 1/2 chance to a bus being late because the results are “late” or “not late”.

1. Two categories alone do not supply their probabilities.
2. A model of the journey or suitable past observations would be needed to estimate those chances.
3. Use 1/2 only when equal chances are supported, rather than inferred from the number of category names.

Check another case

Example 1

A numbered card is selected uniformly from 1–14. Find P(number at least 12).

3/14

12, 13 and 14 are favourable.

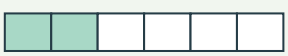
Example 2

Explain what must be established before using favourable outcomes divided by all outcomes.

Define a finite sample space containing every distinct elementary outcome once.

Justify why those elementary outcomes have equal selection chances.

Identify exactly which of them satisfy the event and count using the same outcome unit.



2 of 6

A fair die wins on 5 or 6. What is the chance of losing?

Four other faces lose. The chance is 4/6, or 2/3.

ILLUSTRATED PROBLEM SHEET

Count outcomes exactly once

The counting unit depends on whether order matters and whether an object can be used twice.

1

First	Possible next
A	B or C
B	A or C
C	A or B

Choose a captain then a deputy from three pupils. How many results?

Three choices, then two remaining choices. Six ordered outcomes.

2

AB and BA
↓
Same two-person team
↓
6 ÷ 2 = 3

What if the two roles disappear?

Each unordered pair was counted twice. There are three teams.

3

Die A	Die B
1	5
2	4
3	3
4	2
5	1

How many ordered two-die results give sum six?

Five out of 36 for independent fair dice. The double (3,3) appears once.

4

Tickets 1 to 12
↓
Multiples of 2: 6
↓
Multiples of 3: 4

How many are multiples of 2 or 3?

Subtract the two overlaps, 6 and 12. The union has 6 + 4 − 2 = 8 tickets.

Four pupils in two distinct roles
4 × 3 = 12.

Two-person teams from four
12 ÷ 2 = 6.

CHAPTER PROBLEM

An event has probability 7/12. Find the probability of its complement.

5/12. The event and its complement exhaust all outcomes. Subtract 7/12 from 1 to obtain 5/12.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Can “red” and “blue” be complements in any bag of coloured counters?	Only when every counter is either red or blue. A third colour breaks that claim.
02 Is an unordered pair of die faces always as likely as any other unordered pair?	No. A non-double has two orders; a double has one.
03 A fair six-sided die is rolled. Find P(number less than 3).	1/3 Outcomes 1 and 2 give 2/6 = 1/3.
04 A bag has 5 mint and 7 plum counters, each equally likely to be drawn. Find P(plum).	7/12 Seven plum counters out of twelve total.
05 A card is chosen uniformly from 1–15. Find P(multiple of 4).	1/5 4, 8, 12 give 3/15 = 1/5.
06 P(rain) = 0.35 in a stated forecast model. Find P(no rain).	0.65 1 − 0.35 = 0.65.



One red counter is added to a bag containing two red and two blue. Is it still fair by colour?

No. The new counts are three red and two blue.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 A fair die has faces 1–6. Find P(rolling 9).	0 No face is numbered 9, so the event is impossible.
08 A bag contains only 9 blue counters. One counter is drawn. Find P(blue).	1 Every possible selected counter is blue.
09 A spinner has two labels but one covers three times the area of the other. Can equal label probabilities be assumed?	Two labels alone do not establish equal likelihood. If the spinner is fair by angle and the sectors have the stated 3:1 angle ratio, their probabilities are 3/4 and 1/4. The mechanical behaviour and sector angles must support the model before those probabilities are used.
10 Which number can be a probability? A) 1.4 B) 125% C) 0.72 D) −0.2	0.72 0.72 is between 0 and 1. The others lie outside that interval.
11 P(A) = 9/20. Find P(not A).	11/20 20/20 − 9/20 = 11/20.

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