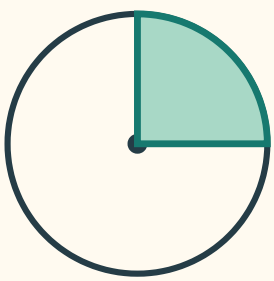
MATHS FIELD GUIDE · GRADE 9

Circular segments through geometric area

OPENING PROBLEM

For radius 8 cm, find the quarter arc.



90°

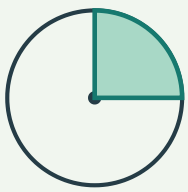
02 Match the fraction of a turn
03 Rules and worked cases
04 A 120° sector uses the same right-triangle pieces
05 From the givens to the answer
06 Choose a base and its perpendicular height
07 Check your reasoning
08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Match the fraction of a turn

For a sector of angle θ in degrees, multiply either the full circumference or full area by θ/360.

1

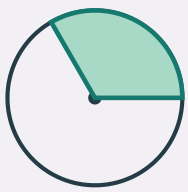


90°

For radius 8 cm, find the quarter arc.

A quarter of 16π cm is 4π cm.

2



120°

For radius 6 cm, find the sector area.

120/360 is one third. A third of 36π is 12π cm².

3



270°

What if the major sector is shaded?

Use the full 270/360 = 3/4. The smaller 90° angle refers to the unshaded part.

4



90°

Find the minor segment area for radius 4 cm.

Quarter disk 4π minus right triangle area 8 gives (4π − 8) cm².

60° sector
one sixth of the disk.

300° sector
five sixths of the disk.

CHAPTER PROBLEM

A 60° segment has radius 11 cm. Find its chord length in cm.

11. The central triangle is equilateral, so its chord equals the radius.

MATHS REFERENCE · GRADE 9

Circular segments through geometric area

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Minor segment at 90°	Radius 8 gives sector area 16π and triangle area 32. Segment area = 16π − 32.
Major segment	Subtract the minor segment from the full disk: 64π − (16π − 32) = 48π + 32.
Semicircle boundary case	A diameter gives a 180° sector and a triangle of zero area, so the segment is half the disk.

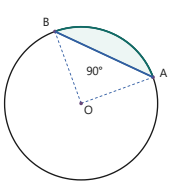
Read the representations

A segment is the part beyond a chord

A chord AB separates a circular segment from the rest of the disk. The minor segment lies between the chord and its minor arc.

For a central angle below 180°, the minor sector consists of the minor segment plus triangle OAB.

Therefore minor-segment area = minor-sector area − area of triangle OAB. A segment is different from a sector.



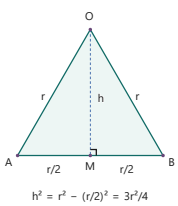
Segment boundary: one chord + one arc

A 60° central triangle is equilateral

OA = OB = r. If angle AOB is 60°, the other two angles share the remaining 120° equally, so all three angles are 60° and AB = r.

Its altitude bisects the base into two lengths r/2. Pythagoras gives $h^2 = r^2 - (r/2)^2 = 3r^2/4$.

Thus $h = r\sqrt{3}/2$ and the equilateral triangle area is $r^2\sqrt{3}/4$. This obtains the needed area without assuming a trigonometric formula.





r = 5

What is the diameter?

Two radii laid end to end through the centre give 10.

THEOREMS AND REPRESENTATIONS

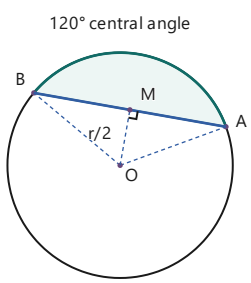
A 120° sector uses the same right-triangle pieces

A 120° sector uses the same right-triangle pieces

The radii form an isosceles triangle with a 120° vertex angle. Its altitude to chord AB also bisects that angle, making two 30°–60°–90° right triangles.

These have the same side proportions as the halves of an equilateral triangle, with the legs interchanged. With hypotenuse r, the height is r/2 and each base half is r√3/2.

The full base is r√3, so the triangle area is (r√3)(r/2)/2 = r²√3/4. Subtract it from the 120° sector area to obtain the minor segment.



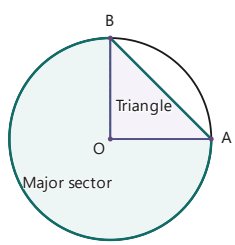
Segment boundary: one chord + one arc

The major segment is what remains of the disk

The same chord bounds a minor and a major segment. Their areas sum to πr².

So major-segment area = disk area − minor-segment area. Equivalently, add the central triangle to the major sector.

The diagram colours the major sector and central triangle separately. Together they make the major segment. Its boundary follows the long arc and the shared chord, not the two radii.



Major segment = major sector + triangle

Boundary: long arc + chord AB

A segment's perimeter has a chord, not two radii

Trace from A to B along the selected arc and return along chord BA. Only those two pieces are exposed.

For a 90° central angle, the chord is the hypotenuse of a right triangle with legs r and r, so its length is r√2.

For a 60° central angle, the chord is r. For a major segment choose the major arc, while the straight chord length is unchanged.



d = 12

Which chord is longest?

A diameter. Its length is twice the radius, here 12.

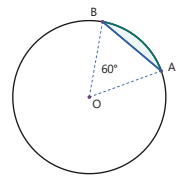
WORKED EXAMPLES

From the givens to the answer

A right central triangle needs no trigonometry

1. A minor segment has radius 4 cm and central angle 90°. Its sector area is (1/4) × 16π = 4π cm².
2. The two radii are perpendicular legs, so triangle OAB has area 4 × 4/2 = 8 cm².
3. The segment area is 4π − 8 cm². The triangle's area is subtracted once, not from the full disk.

Keep both π and the radical in an exact segment area



Segment boundary: one chord + one arc

1. For a 60° segment with radius 6 cm, the sector area is (1/6) × 36π = 6π cm².
2. Its central triangle is equilateral with area 36√3/4 = 9√3 cm².
3. The minor segment area is 6π − 9√3 cm². Leave this exact unless an approximation is requested.

Should triangle area always be subtracted from any sector?

A learner subtracts the central triangle from the major sector to find a major segment.

1. The minor segment is the minor sector minus the triangle.
2. The major segment is the rest of the disk, so it contains the major sector and that triangle.
3. Sketch the chosen region before deciding whether to subtract or add.

Check another case

Example 1

A 60° minor segment has radius 4 cm. Which is its exact area?

8π/3 + 4√3 cm² · 16π − 4√3 cm² · 4π/3 − 8√3 cm² · 8π/3 − 4√3 cm²

8π/3 − 4√3 cm²

The sector is 16π/6 = 8π/3 and the equilateral triangle area is 4√3.

Example 2

A 90° minor segment has radius 12 cm. Which is its exact area?

12π − 72 cm² · 36π − 72 cm² · 36π + 72 cm² · 144π − 72 cm²

36π − 72 cm²

The quarter disk area is 36π and the triangle area is 72.



A radius is 13 and its centre is 5 from a chord. Find the chord length.

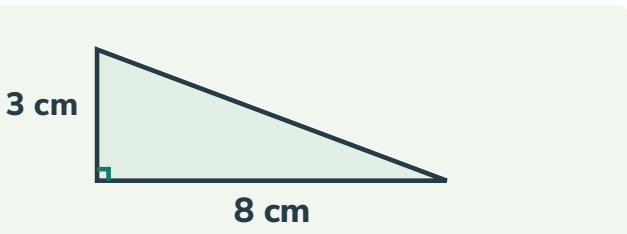
The perpendicular halves the chord. Half-length = √(169 − 25) = 12. Full length = 24.

ILLUSTRATED PROBLEM SHEET

Choose a base and its perpendicular height

Sliding the apex along a line parallel to the base preserves the area.

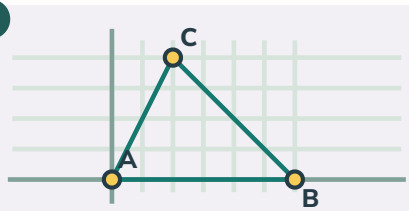
1



What is the area of this right triangle?

Half of an 8 by 3 rectangle, or 12 cm².

2

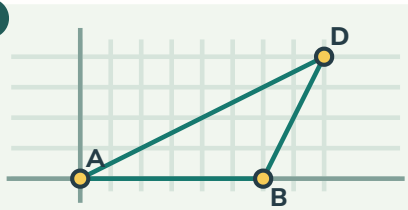


A (0, 0)
B (6, 0)
C (2, 4)

For A(0,0), B(6,0), C(2,4), which height belongs to AB?

The vertical distance is 4. Area = ½ × 6 × 4 = 12 square units.

3



A (0, 0)
B (6, 0)
D (8, 4)

The new apex is D(8,4). Does the area change?

No. The perpendicular foot falls outside AB, but the height to its line is still 4.

4

Area = 30 cm²
↓
Base = 10 cm
↓
Height = 2 × 30 ÷ 10

Find the height without knowing the sloping sides.

The required perpendicular height is 6 cm.

Base doubles, height fixed
area doubles.

Base doubles, height halves
area stays fixed.

CHAPTER PROBLEM

Using π ≈ 3.14, estimate the area of a 90° minor segment of radius 10 cm, in cm².

28.5. The estimate is 25 × 3.14 − 50 = 28.5 cm².

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 A 90° minor segment has radius 2 cm. Which is its exact area? A) $4\pi - 2$ cm² B) $\pi - 4$ cm² C) $\pi - 2$ cm² D) $\pi + 2$ cm²	$\pi - 2$ cm² The quarter sector is π; the right triangle has area 2.
02 Does a sector's perimeter equal its arc length?	Include its two straight radii for a non-full sector. Arc length alone counts only the curved edge.
03 In a circle of radius 6 cm, a chord subtends a minor central angle of 90°. Which is the area of the major segment? A) $9\pi - 18$ cm² B) $36\pi - 18$ cm² C) $27\pi + 18$ cm² D) $27\pi - 18$ cm²	$27\pi + 18$ cm² The minor segment is $9\pi - 18$; subtracting from 36π gives $27\pi + 18$.
04 A minor segment has radius 10 cm and central angle 90°. Which is its perimeter? A) $20\pi + 10$ cm B) $5\pi + 10\sqrt{2}$ cm C) $5\pi + 20$ cm D) $25\pi - 50$ cm	$5\pi + 10\sqrt{2}$ cm The quarter arc is 5π and the chord is $10\sqrt{2}$.
05 A median divides a triangle into two parts. Why are the areas equal?	The two bases on the same line have equal length, and both triangles share the same perpendicular height.
06 For a 120° minor segment of radius r, which triangle area is subtracted? A) $r^2\sqrt{3}/4$ B) $r^2/2$ C) r^2 D) $r^2\sqrt{3}/2$	$r^2\sqrt{3}/4$ The altitude is $r/2$ and the base is $r\sqrt{3}$, so the triangle area is $r^2\sqrt{3}/4$.



Which part is one quarter of the circumference?

The curved arc between the two radii. The filled sector is one quarter of the disk area.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Explain why a 60° central triangle is equilateral.	Its two centre-to-circle sides are equal radii. The remaining angles are equal and sum to 120°, so both are 60°. All three angles are equal, so all three sides are equal and the chord equals r.
08 Why is a segment's perimeter different from a sector's perimeter?	A sector exposes two radial edges and an arc. A segment exposes one chord and its chosen arc. Use the chord length, not 2r, when adding the segment boundary.
09 A minor segment has radius 9 cm and central angle 60°. Its arc length is kπ cm. Find k.	3 The arc is one sixth of 18π, or 3π.
10 Explain two correct ways to find a major segment's area.	Find the minor sector and subtract its central triangle to obtain the minor segment. Subtract that minor segment from the full disk. Alternatively add the same central triangle to the major sector; both expressions agree.
11 A 90° minor segment has radius 8 cm. Which is its exact area? A) 16π − 32 cm² B) 64π − 32 cm² C) 16π + 32 cm² D) 8π − 16 cm²	16π − 32 cm² The quarter sector is 16π and the right triangle area is 32.

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