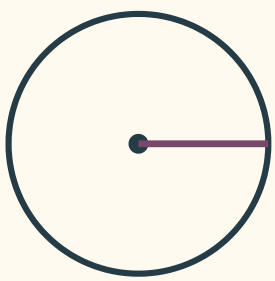
MATHS FIELD GUIDE · GRADE 9

Circle definitions and the unique circumcircle

OPENING PROBLEM

What is the diameter?



r = 5

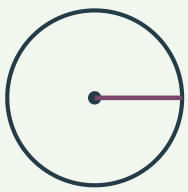
02 Distinguish the radius, chord and arc
03 Rules and worked cases
04 The centre must lie on each perpendicular bisector
05 From the givens to the answer
06 Draw the perpendicular to a chord
07 Check your reasoning
08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Distinguish the radius, chord and arc

The radius goes from the centre to the boundary. A chord has both ends on the boundary.

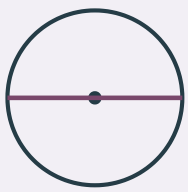
1



What is the diameter?

Two radii laid end to end through the centre give 10.

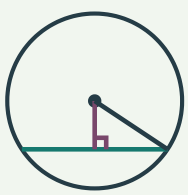
2



Which chord is longest?

A diameter. Its length is twice the radius, here 12.

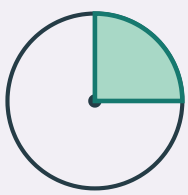
3



A radius is 13 and its centre is 5 from a chord. Find the chord length.

The perpendicular halves the chord. Half-length = $\sqrt{(169 - 25)} = 12$. Full length = 24.

4



Which part is one quarter of the circumference?

The curved arc between the two radii. The filled sector is one quarter of the disk area.

Two distinct points on the circle
a chord joins them.

Three noncollinear points
exactly one circumcircle.

CHAPTER PROBLEM

A circle has radius 9 cm. Find its diameter in cm.

18. The diameter is twice the radius: 18 cm.

MATHS REFERENCE · GRADE 9

Circle definitions and the unique circumcircle

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
One chord	Its perpendicular bisector contains all possible centres of circles through its endpoints.
Three noncollinear points	Two chord bisectors intersect at one point equidistant from all three. It is the unique circumcentre.
Three distinct collinear points	The two relevant bisectors are parallel. No finite circle passes through all three.

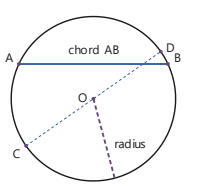
Read the representations

A circle is a set of points at one fixed distance

Every point on a circle is the same distance r from its centre O. That fixed distance is its radius. The filled inside is the circular region.

A chord joins two points on the circle. A diameter is a chord through O; its length is 2r. Every diameter is a chord, but most chords miss the centre.

In the diagram, the dashed diameter and the solid chord have different roles.



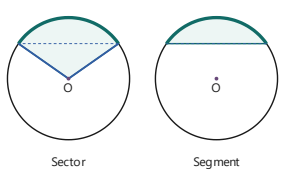
CD passes through O: it is a diameter

An arc is a path; a sector and a segment are regions

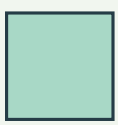
Two distinct points split a circle into two arcs. Unless they are opposite ends of a diameter, one arc is minor and the other is major.

A sector is bounded by two radii and an arc. A segment is bounded by a chord and an arc. The dashed chord in the sector view is only a comparison guide.

A semicircle has a diameter as its straight boundary and half the circle as its arc.



The highlighted boundary arc is the same in both views


square

If a quadrilateral is a square, are its diagonals equal?

Yes. They are hypotenuses of congruent right triangles.

THEOREMS AND REPRESENTATIONS

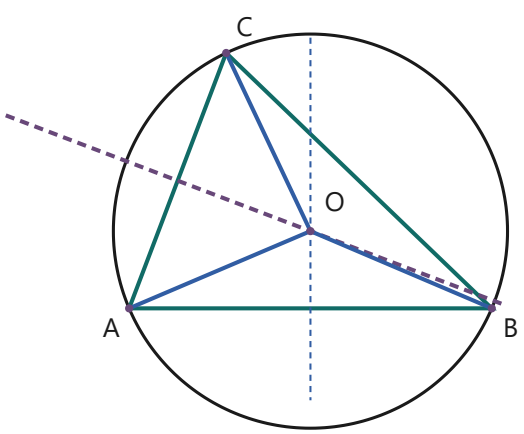
The centre must lie on each perpendicular bisector

The centre must lie on each perpendicular bisector

If a circle goes through A and B, its centre is equally far from them. Therefore it lies on the perpendicular bisector of AB.

For three non-collinear points A, B, C, the perpendicular bisectors of AB and AC meet at O. Their equal-distance properties give OA = OB = OC.

A circle centred at O with radius OA passes through all three points. Any other centre would need to be the same intersection, proving uniqueness.

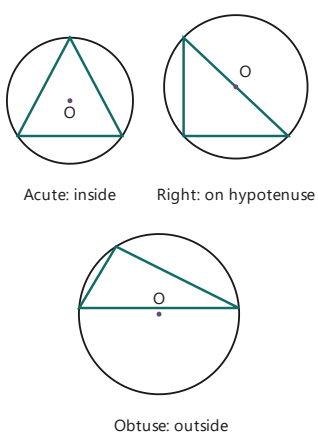


The circumcentre can be inside, on, or outside

For an acute triangle the circumcentre lies inside. For a right triangle it is the midpoint of the hypotenuse.

For an obtuse triangle it lies outside, across the side opposite the obtuse angle. Extend the perpendicular bisectors if necessary.

Each pictured O remains equally far from all three vertices. “Centre” describes the circle, so it need not lie inside the triangle.





rectangle

If the diagonals of a quadrilateral are equal, must it be a square?

A non-square rectangle is a counterexample. Equal diagonals alone do not force equal sides.

WORKED EXAMPLES

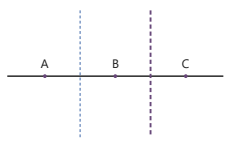
From the givens to the answer

Construct the circumcircle with a compass and ruler

1. Join AB and AC. For AB, draw equal-radius arcs from A and B with radius greater than half AB; join the two arc intersections to make its perpendicular bisector.
2. Repeat for AC. Mark the bisectors' intersection O. Set the compass to OA and draw the circle; check that it passes through B and C as well.
3. This is the circumcircle of △ABC, and O is its circumcentre. O is found from equal distances to vertices, not from the medians or angle bisectors.

Can three points on one line define a circle?

A learner tries to draw a circle through three distinct collinear points.



The perpendicular bisectors are parallel

1. The perpendicular bisectors of two successive segments are parallel distinct lines, so there is no possible centre.
2. A line can intersect a circle at at most two points. Three distinct collinear points therefore cannot all lie on a circle.
3. The unique-circumcircle statement requires three distinct non-collinear points.

Check another case

Example 1

For which triangle is the circumcentre inside?
An acute triangle · Every obtuse triangle · Every right triangle · No triangle

An acute triangle

All three acute angles place the circumcentre inside.

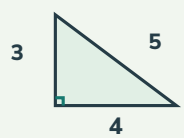
Example 2

Justify uniqueness of the circle through three distinct non-collinear points.

Any centre must lie on the perpendicular bisector of the first pair.

It must also lie on the perpendicular bisector of another pair, and those lines have one intersection.

The intersection and its distance to a given vertex uniquely determine the circle.



Use one diagram to distinguish a theorem from its converse.

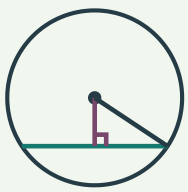
A right angle implies $3^2 + 4^2 = 5^2$ for these lengths. The equality also proves the angle is right by the converse of Pythagoras.

ILLUSTRATED PROBLEM SHEET

Draw the perpendicular to a chord

A perpendicular from the centre to a chord bisects it. The resulting right triangle uses half the chord.

1

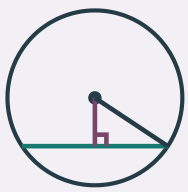


r = 10, d = 6

Find the length of the chord.

Half-length = √(100 − 36) = 8. Double it to get 16.

2

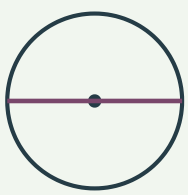


r = 13, AB = 10

How far from the centre is chord AB?

Half the chord is 5. Distance = √(169 − 25) = 12.

3



d = 0

Which chord has its midpoint at the centre?

A diameter. Zero distance leaves half-chord equal to the radius.

4

Radius	Distance	Chord
5	0	10
5	3	8
5	4	6

How does moving a chord away from the centre affect it?

Within the same circle the chord shortens. The right triangle has a fixed hypotenuse.

Radius 5, distance 4
chord length 6.

Radius 5, distance 3
chord length 8.

CHAPTER PROBLEM

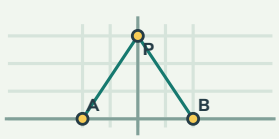
A circular wheel has diameter 27 cm. Find its radius in cm.

27/2. The radius is half the diameter, or 27/2 cm.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Do three distinct points always determine one circle?	Only if they are noncollinear. Their chord bisectors then meet at the unique centre.
02 Which boundary describes a segment of a circle? A) Three straight sides B) Only a radius C) A chord and one of its arcs D) Two radii and an arc	A chord and one of its arcs A segment is the region bounded by a chord and its corresponding arc.
03 Which statement defines a circle? A) All points in a plane at one fixed positive distance from a fixed centre B) All points inside a square C) All points equally far from two fixed centres D) Any closed curved line	All points in a plane at one fixed positive distance from a fixed centre A fixed centre and fixed positive radius determine the circle.
04 Two chords lie 4 cm from the centres of different circles. Must their lengths match?	No. The radii must also match for the half-chord calculations to agree.
05 Which statement about diameters is always true? A) Every chord is a diameter B) A diameter ends at the centre C) A diameter has length r/2 D) Every diameter is a chord	Every diameter is a chord Both diameter endpoints lie on the circle, so it is a chord.
06 Which construction line contains all points equally far from A and B? A) The line AB itself B) Any line parallel to AB C) The perpendicular bisector of AB D) Any line through A	The perpendicular bisector of AB The perpendicular bisector is the equal-distance locus.



A (-2, 0)
P (0, 3)
B (2, 0)

Why is P(0, 3) equidistant from A(−2, 0) and B(2, 0)?

The perpendicular bisector is the y-axis. Both squared distances equal 4 + 9 = 13.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Where is the circumcentre of a right triangle? A) At its right-angle vertex B) Always outside the triangle C) At the intersection of its medians D) At the midpoint of its hypotenuse	At the midpoint of its hypotenuse The hypotenuse midpoint is equally far from all three vertices.
08 How many circles pass through three distinct non-collinear points? A) Infinitely many B) Exactly one C) None D) Exactly two	Exactly one Two perpendicular bisectors determine a unique centre and radius.
09 Why can the circumcentre of an obtuse triangle lie outside the triangle?	The centre is defined by equal distances to the three vertices. The required perpendicular bisectors can meet outside the triangle. A circle about that intersection still passes through all three vertices.
10 Explain why using just the midpoint of AB cannot generally locate the circumcentre of $\triangle ABC$.	The midpoint is only one point on the perpendicular bisector of AB. Equal distance to C is an additional condition. Intersect a second perpendicular bisector to satisfy all three distances.
11 Which region is bounded by two radii and an arc? A) A segment in general B) A triangle only C) A chord D) A sector	A sector Those three boundaries define a sector.

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