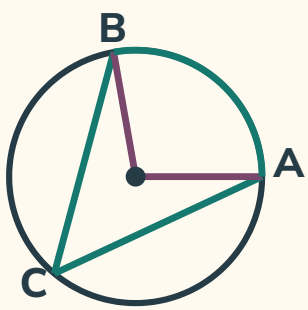
MATHS FIELD GUIDE · GRADE 9

Chords, arcs and central angles

OPENING PROBLEM

A point lies on the opposite arc. What angle does it see at the circumference?



arc 100°

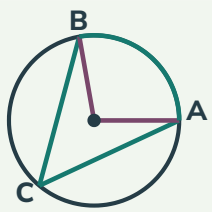
02 Identify which arc the angle sees
03 Rules and worked cases
04 Rotation connects equal arcs and equal chords
05 From the givens to the answer
06 Draw the perpendicular to a chord
07 Check your reasoning
08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Identify which arc the angle sees

A central angle and an inscribed angle must intercept the same arc before using the factor of two.

1

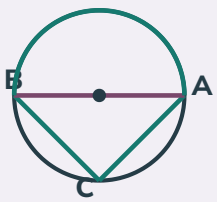


arc 100°

A point lies on the opposite arc. What angle does it see at the circumference?

Half of 100°, so 50°.

2

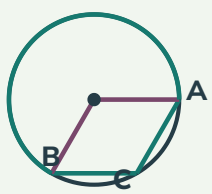


arc 180°

A triangle has this diameter as a side and its third vertex elsewhere on the circle. Find the third angle.

It intercepts a semicircle. Half of 180° is 90°.

3



arc 240°

A vertex on the minor arc sees the major arc. Find its inscribed angle.

Use 240° rather than the minor 120°. The angle is 120°.

4

Arc AB = 140°
↓
Remaining arc = 220°
↓
70° + 110° = 180°

Why are opposite angles of a cyclic quadrilateral supplementary?

Their intercepted arcs together make a full turn. Half of 360° is 180°.

Arc of 80° opposite the vertex
inscribed angle 40°.

Arc of 280° opposite the vertex
inscribed angle 140°.

CHAPTER PROBLEM

A minor arc measures 112°. Find its corresponding major arc measure in degrees.

248°. The major and minor arcs fill one turn. Subtract 112° from 360°.

MATHS REFERENCE · GRADE 9

Chords, arcs and central angles

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Equal chords in one circle	Equal radii plus equal chord give congruent central triangles by SSS, so minor central angles match.
Which arc?	A 70° central angle determines a 70° minor arc and a 290° major arc.
Different circles	Equal central angles alone need not give equal chord lengths. Doubling the radius doubles the matching chord.

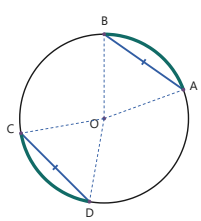
Read the representations

A chord subtends an angle at the centre

Chord AB subtends angle AOB, formed by radii OA and OB. For a non-diameter chord, the smaller angle corresponds to the minor arc.

For two equal chords in the same circle, join both endpoint pairs to O. All four joining lengths are radii.

The two triangles have equal radius pairs and equal chord bases. SSS congruence gives equal minor central angles.

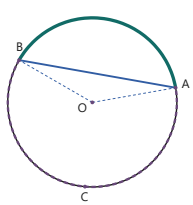


An arc must be identified before its angle is used

A and B determine two arcs. A third point C on the longer route identifies major arc ACB unambiguously.

Arc measure in degrees is the angle swept at the centre along that arc. Minor and major arc measures add to 360°. A semicircular arc measures 180°.

The highlighted short arc and the dashed long arc share endpoints, but have different measures. Arc measure is an angle measure, not a length in centimetres.





r = 5

What is the diameter?

Two radii laid end to end through the centre give 10.

THEOREMS AND REPRESENTATIONS

Rotation connects equal arcs and equal chords

Rotation connects equal arcs and equal chords

In one circle, equal minor central angles let a rotation about O place one radius pair onto the other. Their endpoints and connecting arcs coincide.

So equal minor arcs, equal minor central angles and equal chords are equivalent conditions. Equal chords also have equal corresponding major arcs.

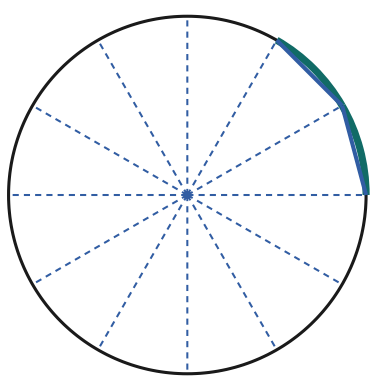
Specify corresponding arcs: a minor arc cannot simply be exchanged for its major partner.

Equal spacing explains wheel spokes and circular motifs

Spokes arranged at equal angles about a wheel's centre divide the rim into equal arcs. Their tips are all one radius from the axle.

Equal neighbouring arcs also give equal chords between neighbouring tips. This is rotation symmetry built from the circle relationships.

Circular motifs such as the Ashoka Chakra use this idea of repeated radial directions. In an ideal wheel, the equal radius also keeps the axle at a constant height while rolling on level ground.



12 equal gaps: 360° ÷ 12 = 30° each



Which chord is longest?

A diameter. Its length is twice the radius, here 12.

WORKED EXAMPLES

From the givens to the answer

The converse uses an included angle

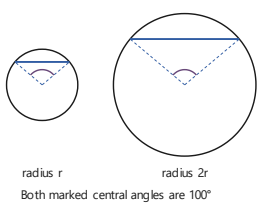
1. Start instead with two equal minor central angles in the same circle. The two adjacent sides in each triangle are radii.
2. SAS makes the two triangles congruent. Their corresponding chord bases are equal.
3. Thus equal chords and equal minor central angles imply each other. The same reasoning works in two circles with equal radii.

Recover the other arc from a full turn

1. A minor arc measures 124°. The major arc with the same endpoints completes the circle.
2. Its measure is 360° − 124° = 236°. The minor central angle is 124°, while the corresponding reflex central angle is 236°.
3. Do not use 180° as the total unless the two arcs together form a semicircle.

Do equal central angles guarantee equal chords in any circles?

A learner compares equal central angles in circles with different radii and claims their chords are equal.



1. The triangles have equal included angles, but their radius pairs are unequal. SAS congruence is unavailable.
2. The larger circle gives the longer chord for this same angle. The figure shows a doubled radius.
3. Use the equal-chord theorem within one circle, or within circles known to have equal radii.

Check another case

Example 1

A semicircular arc has what measure?
360° · 180° · 90° · 270°

180°

It is half a full turn about the centre.

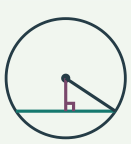
Example 2

Explain the difference between the forward equal-chord theorem and its converse.

The forward theorem starts with equal chords and concludes equal central angles.

The converse starts with equal central angles and concludes equal chords.

Both require one circle or circles with equal radii and corresponding angle choices.



OM ⊥ AB

A radius is 13 and its centre is 5 from a chord. Find the chord length.

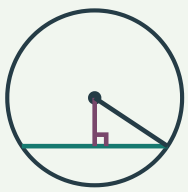
The perpendicular halves the chord. Half-length = √(169 − 25) = 12. Full length = 24.

ILLUSTRATED PROBLEM SHEET

Draw the perpendicular to a chord

A perpendicular from the centre to a chord bisects it. The resulting right triangle uses half the chord.

1

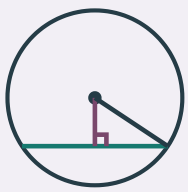


r = 10, d = 6

Find the length of the chord.

Half-length = √(100 − 36) = 8. Double it to get 16.

2

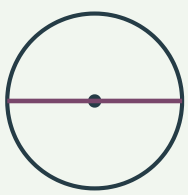


r = 13, AB = 10

How far from the centre is chord AB?

Half the chord is 5. Distance = √(169 − 25) = 12.

3



d = 0

Which chord has its midpoint at the centre?

A diameter. Zero distance leaves half-chord equal to the radius.

4

Radius	Distance	Chord
5	0	10
5	3	8
5	4	6

How does moving a chord away from the centre affect it?

Within the same circle the chord shortens. The right triangle has a fixed hypotenuse.

Radius 5, distance 4
chord length 6.

Radius 5, distance 3
chord length 8.

CHAPTER PROBLEM

A circle has equal chords AB and CD. Minor angle AOB is 76°. Find minor angle COD in degrees.

76°. Equal chords subtend equal minor central angles.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Equal chords AB and CD lie in one circle. Which angles are equal? A) The base angles of every triangle B) Their minor central angles C) Any two angles in the diagram D) Only their reflex angles, never their minor angles	Their minor central angles SSS congruence of the radius triangles gives equal central angles.
02 In one circle, can equal chords have different minor central angles?	No. Triangles formed by the equal chords and two pairs of radii are congruent by SSS.
03 Two chords lie 4 cm from the centres of different circles. Must their lengths match?	No. The radii must also match for the half-chord calculations to agree.
04 A minor arc measures 143°. Find the corresponding major arc measure in degrees.	217° The major and minor arcs fill one turn. Subtract 143° from 360°.
05 A major arc measures 284°. Find its corresponding minor arc measure in degrees.	76° 360 – 284 = 76.
06 Which proof establishes equal chords from equal central angles in one circle? A) SAS with two radius pairs B) SSA with two chord lengths C) AA alone proves congruence D) The drawing looks symmetric	SAS with two radius pairs The equal central angle is included between two pairs of equal radii.



90°

Which part is one quarter of the circumference?

The curved arc between the two radii. The filled sector is one quarter of the disk area.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Equal chords in one circle have which arc relationship? A) Their minor arcs are equal and their major arcs are equal B) Each minor arc equals the other major arc always C) All four arcs measure 180° D) Their arcs have no relationship	Their minor arcs are equal and their major arcs are equal Match minor to minor and major to major.
08 Which extra condition makes equal central angles guarantee equal chords across two circles? A) The circles are drawn in the same colour B) The angles are both acute only C) The circles have equal radii D) The centres lie on a horizontal line	The circles have equal radii Equal radii supply the two equal side pairs for SAS.
09 Prove that equal chords in one circle subtend equal minor central angles.	Join the centre to all four chord endpoints. Two pairs of radii and the given equal chords establish SSS congruence. The corresponding central angles are equal.
10 Why is “arc AB” sometimes an ambiguous instruction?	The same two endpoints determine two routes around a circle. One can be minor and the other major, with different degree measures. Name the minor or major arc, or give an intermediate point to specify the route.
11 Two equal minor arcs in one circle have measures 3x + 5 and 5x − 21 degrees. Find x.	13 3x + 5 = 5x − 21 gives x = 13.

BANANALEARN LEARNING LIBRARY

Where will your maths take you next?

GRADE 9 MATHS

Choose your next problem.

Explore more illustrated guides and practise a related topic.

Browse Grade 9 guides →

Keep exploring

GRADE 9 · TOPIC GUIDE

Chord symmetry and distance from the centre

Open guide →

GRADE 9 · TOPIC GUIDE

Centre, inscribed and semicircle angles

Open guide →

Visit bananalearn.com →