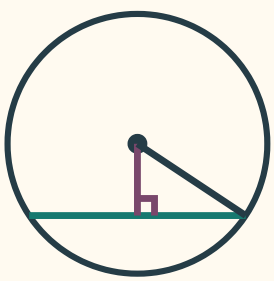
MATHS FIELD GUIDE · GRADE 9

Chord symmetry and distance from the centre

OPENING PROBLEM

Find the length of the chord.



r = 10, d = 6

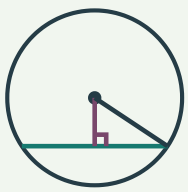
02 Draw the perpendicular to a chord
03 Rules and worked cases
04 From the givens to the answer
05 Distinguish the radius, chord and arc
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Draw the perpendicular to a chord

A perpendicular from the centre to a chord bisects it. The resulting right triangle uses half the chord.

1

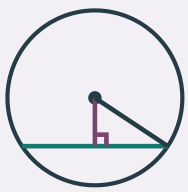


r = 10, d = 6

Find the length of the chord.

Half-length = √(100 − 36) = 8. Double it to get 16.

2

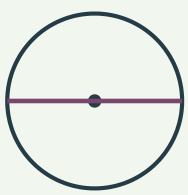


r = 13, AB = 10

How far from the centre is chord AB?

Half the chord is 5. Distance = √(169 − 25) = 12.

3



d = 0

Which chord has its midpoint at the centre?

A diameter. Zero distance leaves half-chord equal to the radius.

4

Radius	Distance	Chord
5	0	10
5	3	8
5	4	6

How does moving a chord away from the centre affect it?

Within the same circle the chord shortens. The right triangle has a fixed hypotenuse.

Radius 5, distance 4
chord length 6.

Radius 5, distance 3
chord length 8.

CHAPTER PROBLEM

A perpendicular from the centre meets a chord of length 24 cm. Find either half in cm.

12. The perpendicular bisects the chord: 24/2 = 12.

MATHS REFERENCE · GRADE 9

Chord symmetry and distance from the centre

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Centre perpendicular to chord	A 24 cm chord splits into halves of 12. At distance 5 cm, radius $\sqrt{(12^2 + 5^2)} = 13$ cm.
Chord length from radius	In a radius-10 circle, distance 6 from centre gives half-chord $\sqrt{(100 - 36)} = 8$, so chord 16.
Compare two chords	In one circle, a chord nearer the centre has a larger half-chord. At distance zero the chord is a diameter.

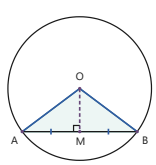
Read the representations

The distance to a chord is perpendicular

The distance from O to chord AB means the length of the perpendicular from O to the line AB. For a circle chord, its foot M lies on the segment.

For a non-diameter chord, the radii OA and OB are equal. With OM shared and right angles at M, $\triangle OMA \cong \triangle OMB$ by RHS.

Thus AM = MB: a perpendicular from the centre bisects the chord. For a diameter, the perpendicular foot is O itself, already its midpoint.



Compare △OMA and △OMB by RHS

A chord's perpendicular bisector passes through the centre

The forward and converse results describe the same symmetry: the perpendicular bisector of every chord contains O.

For a diameter, its perpendicular bisector is the line through O perpendicular to that diameter. This statement still works even though the midpoint-to-centre segment has zero length.

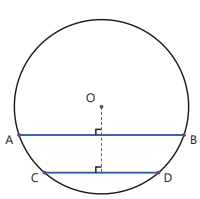
For a non-diameter chord, the two right triangles also give (half chord)^2 + (centre distance)^2 = r^2.

Nearer the centre means a longer chord

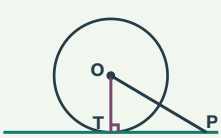
In one circle, compare two chords using perpendicular distances d_1 and d_2. Each has half-length squared $r^2 - d^2$.

A smaller distance leaves a larger half-length squared, so the nearer chord is longer. Equal distances give equal chords, and equal chords give equal distances.

The diameter has distance zero and length 2r, so it is the longest chord. A line at distance greater than r does not meet the circle.



Compare perpendicular distances, not sloping lengths



OT ⊥ PT

OT = 5 cm and OP = 13 cm. How long is PT?

OP is the hypotenuse. PT = $\sqrt{(169 - 25)} = 12$ cm.

WORKED EXAMPLES

From the givens to the answer

Reverse the givens to prove the converse

1. Suppose M is the midpoint of a chord AB that is not a diameter. Join OA, OB and OM.
2. OA = OB, AM = MB and OM is shared, so △OMA ≅ △OMB by SSS. Hence angles OMA and OMB are equal.
3. These adjacent angles form a straight angle, so each is 90°. The line from the centre to the midpoint is perpendicular to the chord. For a diameter, the midpoint is O itself, so OM is not a line.

Find a chord by first finding its half

1. A circle has radius 10 cm. The perpendicular from the centre to a chord has length 6 cm.
2. The perpendicular bisects the chord. If its half-length is h, then $h^2 = 10^2 - 6^2 = 64$, so h = 8 cm.
3. The full chord is 16 cm. The positive root is required because h is a length.

Can a sloping centre-to-chord segment be used as the distance?

A learner measures a sloping segment from O to one chord endpoint and calls it the chord's distance.

1. A centre-to-endpoint segment is a radius. It is not generally perpendicular to the chord.
2. The distance is measured to the perpendicular foot, which is the midpoint of a non-diameter chord.
3. Mark the right angle before using Pythagoras or comparing chord distances.

Check another case

Example 1

M is the midpoint of a non-diameter chord AB. What is angle OMA?

180° · 90° · 45° · 60°

90°

The line joining O to the midpoint is perpendicular to the chord.

Example 2

Why is a diameter the longest chord of a given circle?

Every chord half has square $r^2 - d^2$.

The smallest possible perpendicular distance d is zero.

This gives half-length r and full length 2r, attained by a diameter.



r = 5, d = 3

How can distance to a line distinguish a secant from a tangent?

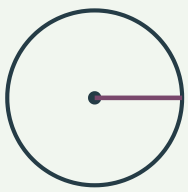
Three is less than the radius five. The line crosses twice. Half the chord is four.

ILLUSTRATED PROBLEM SHEET

Distinguish the radius, chord and arc

The radius goes from the centre to the boundary. A chord has both ends on the boundary.

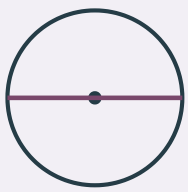
1



What is the diameter?

Two radii laid end to end through the centre give 10.

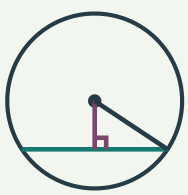
2



Which chord is longest?

A diameter. Its length is twice the radius, here 12.

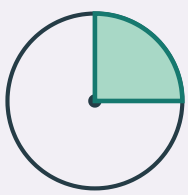
3



A radius is 13 and its centre is 5 from a chord. Find the chord length.

The perpendicular halves the chord. Half-length = $\sqrt{(169 - 25)} = 12$. Full length = 24.

4



Which part is one quarter of the circumference?

The curved arc between the two radii. The filled sector is one quarter of the disk area.

Two distinct points on the circle
a chord joins them.

Three noncollinear points
exactly one circumcircle.

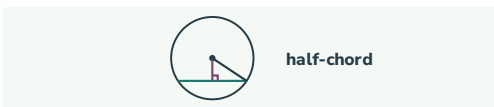
CHAPTER PROBLEM

A circle has radius 13 cm and a chord at perpendicular distance 5 cm. Find its full length in cm.

24. Half length squared = 169 − 25 = 144; the half is 12, so full length is 24 cm.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Two chords lie 4 cm from the centres of different circles. Must their lengths match?	No. The radii must also match for the half-chord calculations to agree.
02 Do three distinct points always determine one circle?	Only if they are noncollinear. Their chord bisectors then meet at the unique centre.
03 A circle has radius 17 cm. A chord is 8 cm from the centre. Find its full length in cm. 	30 Half length = √(289 − 64) = 15, giving 30 cm.
04 A chord is 16 cm long in a circle of radius 10 cm. Find its perpendicular distance from the centre in cm.	6 The half-chord is 8; distance squared = 100 − 64 = 36, so distance is 6 cm.
05 Two chords in one circle are at distances 4 cm and 7 cm from O. Which is longer? A) The information can never compare them B) The chord at 4 cm C) The chord at 7 cm D) They must be equal	The chord at 4 cm In the same circle, the chord nearer the centre is longer.
06 Two equal chords of one circle have perpendicular distances p and q from its centre. Which follows? A) p = 2q B) p + q = r always C) p and q must both be zero D) p = q	p = q Equal chords are equidistant from the centre.

PA = PB
↓
3x − 2 = x + 8
↓
x = 5

Tangents from one external point have these expressions. Find their lengths.

Both equal 13. They run from the same point to different contacts on the same circle.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 A circle has radius 11 cm. Find the greatest possible chord length in cm.  	22 The diameter is the longest chord: 2 × 11 = 22 cm.
08 A line is 9 cm from the centre of a circle of radius 7 cm. How many intersection points are there? A) Zero B) One C) Two D) Infinitely many	Zero Its distance is greater than the radius, so it misses the circle.
09 Prove that a perpendicular from a circle's centre bisects a non-diameter chord.	Join the centre to both endpoints to form equal radii. The two right triangles have equal hypotenuses and a shared perpendicular leg, so they are congruent by RHS. Corresponding chord halves are equal.
10 Explain why comparing chord distances in two circles of different radii is insufficient to compare chord lengths.	A half-chord satisfies $h^2 = r^2 - d^2$. Both the radius and the perpendicular distance affect h. The nearer-is-longer comparison requires one circle or equal radii.
11 A circle has radius 25 cm and a chord 7 cm from its centre. Find the chord length in cm.  	48 The half is √(625 − 49) = 24, so the chord is 48 cm.

$\angle OAP = \angle OBP = 90°$
↓
$\angle APB = 54°$
↓
$\angle AOB = 126°$

Why do the last two angles sum to 180°?

The right angles already use 180° of quadrilateral OAPB's 360°.

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