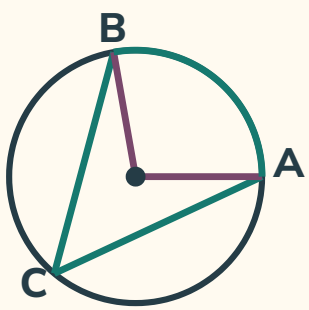
MATHS FIELD GUIDE · GRADE 9

Centre, inscribed and semicircle angles

OPENING PROBLEM

A point lies on the opposite arc. What angle does it see at the circumference?



arc 100°

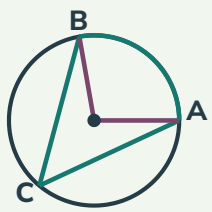
02 Identify which arc the angle sees
03 Rules and worked cases
04 Subtract when both endpoints lie on one side of PC
05 From the givens to the answer
06 Distinguish the radius, chord and arc
07 Check your reasoning
08 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Identify which arc the angle sees

A central angle and an inscribed angle must intercept the same arc before using the factor of two.

1

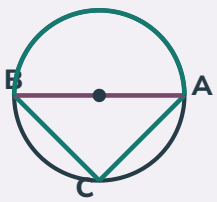


arc 100°

A point lies on the opposite arc. What angle does it see at the circumference?

Half of 100°, so 50°.

2

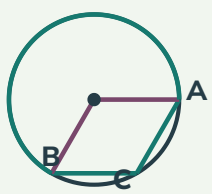


arc 180°

A triangle has this diameter as a side and its third vertex elsewhere on the circle. Find the third angle.

It intercepts a semicircle. Half of 180° is 90°.

3



arc 240°

A vertex on the minor arc sees the major arc. Find its inscribed angle.

Use 240° rather than the minor 120°. The angle is 120°.

4

Arc AB = 140°
↓
Remaining arc = 220°
↓
70° + 110° = 180°

Why are opposite angles of a cyclic quadrilateral supplementary?

Their intercepted arcs together make a full turn. Half of 360° is 180°.

Arc of 80° opposite the vertex
inscribed angle 40°.

Arc of 280° opposite the vertex
inscribed angle 140°.

CHAPTER PROBLEM

An inscribed angle intercepts an arc of 104°. Find the angle in degrees.

52°. An inscribed angle measures half its intercepted arc: 104° ÷ 2 = 52°.

MATHS REFERENCE · GRADE 9

Centre, inscribed and semicircle angles

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Same intercepted arc	A central angle of 104° gives an inscribed angle of 52° when its vertex is on the other arc.
Vertex on the minor arc	If the minor arc is 104°, the vertex intercepts the 256° major arc. The inscribed angle is 128°.
Diameter	Its arc is 180°, so the angle subtended at another point of the circle is 90°.

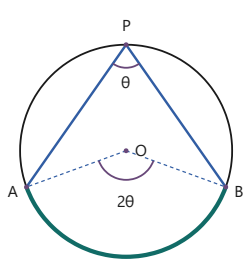
Read the representations

An inscribed angle has its vertex on the circle

An inscribed angle APB has its vertex P on the circle and its arms along chords PA and PB.

It intercepts the arc from A to B that does not contain P. The angle at the centre measured along that particular arc is twice angle APB.

Trace the highlighted arc before deciding whether the central angle is minor or reflex.



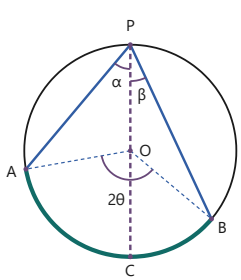
Highlighted arc: 140°; angle APB: 70°

Add the two diameter cases when C lies inside the angle

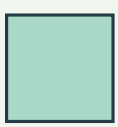
Draw diameter PC through the inscribed-angle vertex. In this arrangement PC splits angle APB into α = angle APC and β = angle CPB. Thus θ = α + β.

The preceding proof doubles each smaller inscribed angle. Adding gives the central arc angle AOC + COB = 2(APC + CPB).

Thus the central angle on the intercepted arc AB is twice angle APB.



Highlighted arc: 130°; angle APB: 65°



square

If a quadrilateral is a square, are its diagonals equal?

Yes. They are hypotenuses of congruent right triangles.

THEOREMS AND REPRESENTATIONS

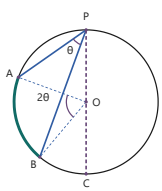
Subtract when both endpoints lie on one side of PC

Subtract when both endpoints lie on one side of PC

Here C is outside angle APB, so angle APB = angle APC − angle BPC.

The diameter proof gives central angle AOC = 2APC and BOC = 2BPC. Subtract the smaller from the larger.

The central angle AOB on the intercepted arc is therefore 2(APC − BPC) = 2APB. The rule covers both arrangements.



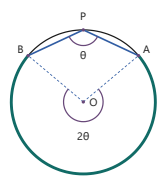
Highlighted arc 70°; angle APB: 35°

The intercepted arc can be the longer route

When P lies on the minor arc AB, angle APB intercepts the major arc. The corresponding central angle is reflex.

The same doubling result applies. One way to see this case is to add the two diameter results across the major arc.

Using the smaller central angle automatically would give the wrong inscribed angle.



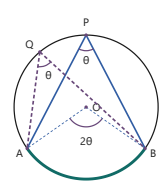
Highlighted arc 260°; angle APB: 130°

The same segment means the same side of the chord

Points P and Q on the same arc of chord AB intercept the same other arc. Each inscribed angle is half its central angle.

Therefore angles APB and AQB are equal. This is the angles-in-the-same-segment theorem.

Points on opposite sides of AB intercept arcs that complete the circle; their angles are supplementary instead.



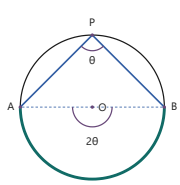
Highlighted arc 110°; angle APB: 55°

A diameter gives a right angle on the circle

A diameter intercepts a semicircular arc of 180°. An angle on the remaining semicircle is half of that, so it is 90°.

Conversely, if angle APB is 90° with A, P and B on a circle, its intercepted arc is 180°, so AB is a diameter.

The right-angle vertex must be on the circle; a point anywhere inside the region is insufficient.



Highlighted arc 180°; angle APB: 90°



rectangle

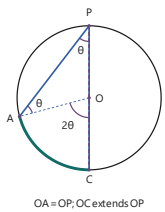
If the diagonals of a quadrilateral are equal, must it be a square?

A non-square rectangle is a counterexample. Equal diagonals alone do not force equal sides.

WORKED EXAMPLES

From the givens to the answer

Begin the proof with a diameter through the vertex



OA = OP; OC extends OP

1. Let PC be a diameter through P and take another point A. OA = OP, so the base angles OAP and APO of △AOP are equal; call each θ.
2. OC extends OP beyond O. The exterior angle AOC equals the two remote interior angles, θ + θ = 2θ.
3. Since PC is along PO, angle APC = θ. Hence the central angle AOC is twice the inscribed angle APC on the same arc AC.

Which central angle should be halved?

A learner halves a 140° minor central angle even though P lies on its minor arc.

1. The intercepted arc must avoid P, so it is the major arc of 220°.
2. The inscribed angle is 220°/2 = 110°, not 70°.
3. Choose the arc first, then halve its corresponding central angle.

Check another case

Example 1

Prove that an angle subtended by a diameter at another point on its circle is a right angle.

The diameter endpoints determine a semicircular arc of 180°.

The inscribed angle intercepts that semicircle not containing its vertex.

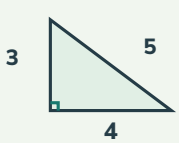
The centre-angle theorem gives half of 180°, which is 90°.

Example 2

An inscribed angle intercepts an arc of 154°. Find the angle in degrees.

77°

154/2 = 77.



Use one diagram to distinguish a theorem from its converse.

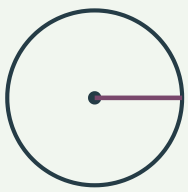
A right angle implies $3^2 + 4^2 = 5^2$ for these lengths. The equality also proves the angle is right by the converse of Pythagoras.

ILLUSTRATED PROBLEM SHEET

Distinguish the radius, chord and arc

The radius goes from the centre to the boundary. A chord has both ends on the boundary.

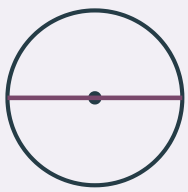
1



What is the diameter?

Two radii laid end to end through the centre give 10.

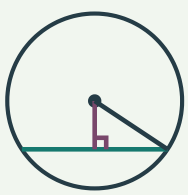
2



Which chord is longest?

A diameter. Its length is twice the radius, here 12.

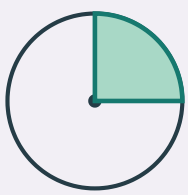
3



A radius is 13 and its centre is 5 from a chord. Find the chord length.

The perpendicular halves the chord. Half-length = $\sqrt{(169 - 25)} = 12$. Full length = 24.

4



Which part is one quarter of the circumference?

The curved arc between the two radii. The filled sector is one quarter of the disk area.

Two distinct points on the circle
a chord joins them.

Three noncollinear points
exactly one circumcircle.

CHAPTER PROBLEM

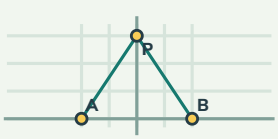
An inscribed angle is 38°. Find the measure of its intercepted arc in degrees.

76°. The intercepted arc has twice the measure of the inscribed angle: $2 \times 38° = 76°$.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 In one circle, can equal chords have different minor central angles?	No. Triangles formed by the equal chords and two pairs of radii are congruent by SSS.
02 Do three distinct points always determine one circle?	Only if they are noncollinear. Their chord bisectors then meet at the unique centre.
03 An inscribed angle is 116°. Find its intercepted arc measure in degrees.	232° The intercepted arc has twice the measure of the inscribed angle: 2 × 116° = 232°.
04 Minor arc AB is 86°. P lies on that minor arc. Find angle APB in degrees.	137° P intercepts the major arc, 360 − 86 = 274°, so angle APB = 137°.
05 P and Q lie on the same side of chord AB on one circle. Angle APB is 57°. Find angle AQB in degrees.	57° Angles in the same segment are equal.
06 AB is a diameter and P is another point on the circle. Find angle APB in degrees.	90° The semicircular arc is 180°, so the angle is 90°.



A (-2, 0)
P (0, 3)
B (2, 0)

Why is P(0, 3) equidistant from A(−2, 0) and B(2, 0)?

The perpendicular bisector is the y-axis. Both squared distances equal 4 + 9 = 13.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 A, P and B lie on a circle and angle APB = 90°. What follows? A) PB is necessarily a diameter B) O must equal P C) AB is a diameter D) AP is necessarily a diameter	AB is a diameter The intercepted arc measures 180°, so AB joins opposite circle points.
08 Use a diameter PC to explain why the centre-angle theorem sometimes needs subtraction.	A and B can lie on the same side of the diameter line. Then angle APB is the difference of angles APC and BPC, not their sum. The corresponding central angles are each twice those angles, so their difference is twice APB.
09 Why do two circle points on opposite sides of chord AB generally give supplementary, rather than equal, angles subtending AB?	Each angle intercepts the arc on the other side from its own vertex. The two intercepted arc measures add to 360°. Halving them makes the two inscribed angles total 180°.
10 Minor arc XY is 128°. P lies on the major arc XY. Find angle XPY in degrees.	64° P intercepts the minor arc of 128°, giving 64°.
11 An inscribed angle intercepts an arc of 246°. Find the inscribed angle in degrees.	123° An inscribed angle measures half its intercepted arc: 246° ÷ 2 = 123°.

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