MATHS FIELD GUIDE · GRADE 9

Arithmetic sequences and the nth term

OPENING PROBLEM

What do the four gaps show?

7 11 15 19 23

02 Distinguish a term from a total
03 Rules and worked cases
04 From the givens to the answer
05 Track the jump and the starting point
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Distinguish a term from a total

In an arithmetic progression, $a_n = a_1 + (n - 1)d$.

1

7 11 15 19 23

What do the four gaps show?

The common difference is 4. Reaching term 5 takes four jumps.

2

$a_1 = 7$ and $d = 4$

↓

$a_{12} = 7 + 11 \times 4$

↓

51

Why use eleven instead of twelve?

The first term is already present before making any jumps.

3

$7 + 4(n - 1) = 39$

↓

$4(n - 1) = 32$

↓

$n = 9$

Does 39 occur?

Yes. The position is a positive integer. The adjacent terms are 35 and 43.

4

Position	3	8
Term	15	35

Recover the difference without knowing term 1.

Five gaps add 20, so d = 4. Step back twice from 15 to get 7.

d = 0
a constant AP.

d < 0
each term is smaller than its predecessor.

CHAPTER PROBLEM

Find the common difference of 18, 14, 10, … .

-4. 14 − 18 = −4.

MATHS REFERENCE · GRADE 9

Arithmetic sequences and the nth term

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Forward calculation	a = 12 and d = −3 give term 7 = 12 + 6(−3) = −6.
Recover d	Terms 3 and 8 are 11 and 31. Five equal jumps span 20, so d = 4.
Membership	For 2, 7, 12, ..., solve 2 + 5(n − 1) = 43. n is not a whole number, so 43 is absent.

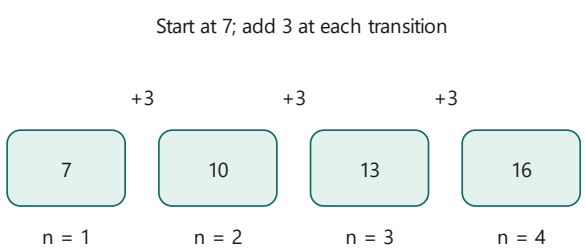
Read the representations

An arithmetic sequence adds a fixed difference

The common difference d is the next term minus the previous term. It can be positive, zero or negative.

For 7, 10, 13, 16, ..., the first term is a = 7 and d = 3.

The gaps are equal even though the term values change.



From the first term to position n there are n − 1 equal changes.

Count the changes, not just the position

Term 1 is a. Term 2 is a + d. Term 3 is a + 2d.

Thus the nth term is $a_n = a + (n - 1)d$ for n = 1, 2, 3, ...

Substituting n = 1 must recover the first term. This checks the indexing.

Forward	2	5	8	11
Reverse	11	8	5	2
Pair	13	13	13	13

Find the sum of the forward row.

Four pairs give 52, twice the sum. The sum is 26.

WORKED EXAMPLES

From the givens to the answer

Should the rule use nd?

A learner writes $a_n = a + nd$.

1. At n = 1 that expression gives a + d, already the second term.
2. Only n − 1 changes happen after term 1.
3. The correct rule is a + (n − 1)d.

Find the 12th term of 4, 9, 14, ...

1. Here a = 4 and d = 5.
2. $a_{12} = 4 + 11 \times 5 = 59$.
3. The position is 12; the term value is 59.

Is 40 a term of 5, 12, 19, ...?

1. Set 5 + (n − 1)7 = 40.
2. Then 7(n − 1) = 35, so n = 6.
3. Because 6 is a positive integer position, 40 is the sixth term.

Check another case

Example 1

How many gaps separate the first and ninth terms?

7 · 8 · 9 · 10

8

There are 9 − 1 = 8 gaps.

Example 2

Derive $a_n = a + (n - 1)d$ in your own words.

Begin with value a at position 1.

Each move to the next position adds d.

Reaching position n makes n − 1 moves, giving the formula.

$$1 + 2 + \cdots + 20$$
$$\downarrow$$
$$20 \times 21 \div 2$$
$$\downarrow$$
$$210$$

Which two terms form the first pair?

1 and 20. Every pair of opposite positions totals 21.

ILLUSTRATED PROBLEM SHEET

Track the jump and the starting point

A position counts terms. From the first to the nth term there are n − 1 gaps.

1

3 7 11 15 ?

What is the next term if the rule is add four?

Nineteen. Each gap is four.

2

Position	1	2	3	4
Term	3	7	11	15

How do you reach term 4 straight from term 1?

Start at 3 and add three jumps of 4. The result is 15.

3

22 17 12 7 ?

Can the jump be negative?

Yes. Subtract five each time. The next term is 2.

4

Input	1	2	3
Output	3	5	7

Which rule doubles then adds one?

Output = 2 × input + 1. Input 6 gives 13.

Rule “add six”, start two
term four is 20.

Rule “multiply by six”, start two
term four is 432.

CHAPTER PROBLEM

Find the fifth term when a = 2 and d = 6.

26. 2 + 4 × 6 = 26.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Is 40 a term of this progression?	No. Its difference from 7 is 33, which is not a whole number of four-unit gaps.
02 Do the first two numbers 2, 4 fix the whole sequence?	No. Adding two gives 6 next. Doubling gives 8 next. A rule is needed.
03 Find the tenth term of 3, 7, 11, … .	39 $3 + 9 \times 4 = 39$.
04 Which rule generates 6, 11, 16, …? A) 5n + 1 B) 6n + 5 C) 5n + 6 D) n + 5	5n + 1 $6 + (n - 1)5 = 5n + 1$.
05 Find the eighth term when a = 30 and d = −3.	9 $30 + 7(-3) = 9$.
06 Which position contains 35 in 5, 10, 15, …?	7 The rule is 5n; 5n = 35 gives n = 7.

A term lies halfway between two others

Term 3 is 11 and term 11 is 43. Eight jumps add 32, so each jump adds four. Term 7 is four jumps from term 3 and equals 27.

The terms on either side of term 7 have matching gaps. The average of 11 and 43 is also 27.

4 + 7 + 10 + 13 + 16
↓
2 × 20 + 10
↓
50

What if one middle term is left over?

Add the two outer pairs and then the middle 10. The average is 10.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Is 24 a term of 2, 7, 12, ...? A) Yes, fifth B) Yes, sixth C) No, the position would not be an integer D) No, arithmetic sequences cannot reach 24	No, the position would not be an integer $2 + 5(n - 1) = 24$ gives $n = 5.4$, which is not a position.
08 An arithmetic sequence has first term 9 and third term 21. Find d.	6 Two equal gaps total 12, so $d = 6$.
09 Explain why n must be a positive integer in an ordinary nth-term sequence rule.	n counts positions starting at 1. Fractional or negative position numbers are not included in that indexing. An algebraic formula may give other outputs, but they are not terms of this sequence.
10 Explain why an arithmetic sequence can be constant.	The common difference may be zero. Adding zero preserves the same value at every step. The formula gives $a_n = a$ for all positions.
11 Find the sixth term when $a = -2$ and $d = 4$.	18 $-2 + 5 \times 4 = 18$.

$S_{10} = 190$
↓
$S_9 = 153$
↓
$a_{10} = 37$

How can two totals reveal just one term?

The first nine terms occur in both totals. Subtraction leaves the tenth alone.

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