MATHS FIELD GUIDE · GRADE 9

Area from three sides

OPENING PROBLEM

What is s and why is the area six?

Sides	3	4	5
s minus side	3	2	1

02 Recover area from the side lengths
03 Rules and worked cases
04 From the givens to the answer
05 Choose a base and its perpendicular height
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Recover area from the side lengths

First test the triangle inequalities. For a triangle, $s = (a + b + c)/2$ and $\text{area}^2 = s(s - a)(s - b)(s - c)$.

1

Sides	3	4	5
s minus side	3	2	1

What is s and why is the area six?

s = 6. The product 6 × 3 × 2 × 1 is 36. Its positive square root is 6.

2

Sides 5, 5, 6

↓

s = 8

↓

$\text{Area}^2 = 8 \times 3 \times 3 \times 2$

How can symmetry check the answer?

Area is 12. Bisect the base into two lengths of 3. The altitude is 4, so ½ × 6 × 4 = 12.

3

Sides 13, 14, 15

↓

s = 21

↓

$\text{Area}^2 = 21 \times 8 \times 7 \times 6$

Find the altitude to the side of 14.

The product is 7056. Area is 84, so height is 2 × 84 ÷ 14 = 12.

4

2 **3** **5**

Can these lengths enclose positive area?

No. The two shorter lengths add exactly to the longest. The zero Heron product reflects a collapsed shape.

Sides 5, 12, 13
area 30.

Sides 5, 5, 10
zero area, a collapsed configuration.

CHAPTER PROBLEM

Find the semiperimeter of a triangle with sides 9, 10 and 11 cm, in cm.

15. The perimeter is 30, so s = 15.

MATHS REFERENCE · GRADE 9

Area from three sides

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Heron's formula	For sides 5, 5, 6, semiperimeter s = 8. Area √(8 × 3 × 3 × 2) = 12.
Independent check	Split the 6-unit base into two 3s. The altitude is √(25 − 9) = 4. Area = 6 × 4/2 = 12.
First check feasibility	For 2,3,6, two shorter sides sum to less than six. No triangle exists, so no area is defined.

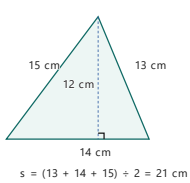
Read the representations

Three sides can replace a missing height

Heron's formula uses the three side lengths a, b and c. First calculate the semiperimeter s = (a + b + c)/2.

The area is √[s(s − a)(s − b)(s − c)]. All lengths must use the same unit. The positive square root gives area.

For the pictured triangle, the side data also give a base-height calculation. Both methods must agree.



Check whether the triangle exists first

The sides must be positive, and the sum of any two must exceed the third. After ordering positive sides, it is enough to test the two shorter against the longest.

Equality produces a flattened, zero-area boundary case rather than an ordinary triangle. A smaller sum is impossible.

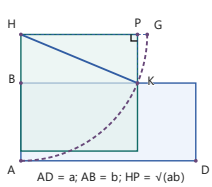
A calculator result cannot turn impossible side lengths into a valid triangle.

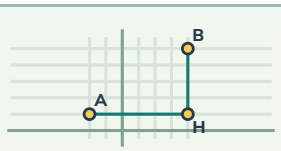
Squaring a rectangle preserves its area, not its perimeter

To square a shape means to construct a square of equal area. The figure uses a form of Baudhāyana's rectangle construction.

Start with rectangle AD × AB = a × b, where a > b. Put R = (a + b)/2. Take AH = HG = R and draw the circular arc from A to G about H. It meets the line BC at K.

Draw KP perpendicular to HG. The square with side HP is the required equal-area square. The shaded square is inside the construction.





A (-2, 1)
H (4, 1)
B (4, 5)

Find the direct distance from A(−2, 1) to B(4, 5).

The legs are 6 and 4. AB = √52 = 2√13. The broken route via H is 10 units.

WORKED EXAMPLES

From the givens to the answer

Keep the factors separate until they are checked

1. For sides 13, 14 and 15 cm, s = 21 cm. The four factors are 21, 8, 7 and 6.
2. Their product is 7056 cm⁴. Its square root is 84 cm². Using base 14 cm and perpendicular height 12 cm also gives 84 cm².
3. The perimeter is 42 cm, not 84 cm². A square root changes the fourth-power length unit into a square unit.

Check Heron using a perpendicular height

1. An isosceles triangle has sides 5, 5 and 6 cm. Here s = 8, so Heron gives $\sqrt{(8 \times 3 \times 3 \times 2)} = 12$ cm².
2. The altitude bisects the base into lengths 3 and 3. Pythagoras gives height $\sqrt{(25 - 9)} = 4$ cm.
3. The base-height method also gives $6 \times 4/2 = 12$ cm². For a non-square product, take out a square factor: $\sqrt{20} = \sqrt{(4 \times 5)} = 2\sqrt{5}$, since $(2\sqrt{5})^2 = 20$. Keep an exact radical instead of silently rounding.

Pythagoras verifies the rectangle construction

1. HK = R because K lies on the arc. Also PK = AH − AB = R − b = (a − b)/2. Triangle HPK is right-angled at P.
2. Thus $HP^2 = R^2 - PK^2 = [(a + b)^2 - (a - b)^2]/4 = ab$.
3. A square of side HP therefore has exactly the rectangle's area ab. For a = 9 and b = 4, the side is 6. Equal area does not require equal perimeters.

Can the perimeter be substituted for s?

A learner uses a + b + c in every place where Heron's formula has s.

1. The symbol s denotes half the perimeter.
2. Calculate s once, then form the three differences s − a, s − b and s − c.
3. Check the triangle inequalities before multiplying; taking an absolute value of an invalid product is not a repair.

Check another case

Example 1

Explain why the rectangle construction gives an equal-area square.

The right triangle HPK has hypotenuse (a + b)/2 and other leg (a − b)/2.

Pythagoras makes HP² the difference of their squares, which simplifies to ab.

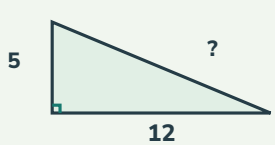
A square with side HP has area ab, the same as the original rectangle.

Example 2

Find the area of a triangle with sides 9, 12 and 15 cm in cm².

54

s = 18; $\sqrt{(18 \times 9 \times 6 \times 3)} = 54$.



A right triangle has legs 5 and 12. How long is its hypotenuse?

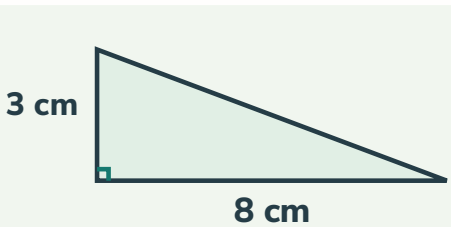
$5^2 + 12^2 = 169$, so the longest side is 13.

ILLUSTRATED PROBLEM SHEET

Choose a base and its perpendicular height

Sliding the apex along a line parallel to the base preserves the area.

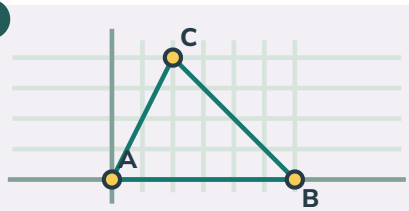
1



What is the area of this right triangle?

Half of an 8 by 3 rectangle, or 12 cm².

2

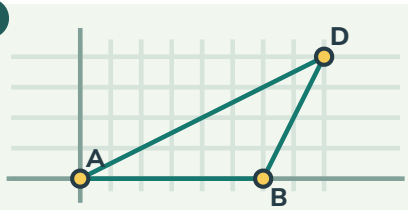


A (0, 0)
B (6, 0)
C (2, 4)

For A(0,0), B(6,0), C(2,4), which height belongs to AB?

The vertical distance is 4. Area = ½ × 6 × 4 = 12 square units.

3



A (0, 0)
B (6, 0)
D (8, 4)

The new apex is D(8,4). Does the area change?

No. The perpendicular foot falls outside AB, but the height to its line is still 4.

4

Area = 30 cm²
↓
Base = 10 cm
↓
Height = 2 × 30 ÷ 10

Find the height without knowing the sloping sides.

The required perpendicular height is 6 cm.

Base doubles, height fixed
area doubles.

Base doubles, height halves
area stays fixed.

CHAPTER PROBLEM

Use Heron's formula for sides 6, 8 and 10 cm. Find the area in cm².

24. s = 12; area = √(12 × 6 × 4 × 2) = 24.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

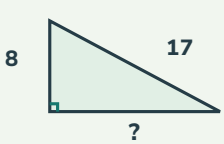
Check your reasoning

Try the problem	Solution and reasoning
01 If all side lengths double, how does Heron's formula reflect area scaling?	All four factors double. The product grows sixteenfold; its square root grows fourfold.
02 A median divides a triangle into two parts. Why are the areas equal?	The two bases on the same line have equal length, and both triangles share the same perpendicular height.
03 Find the area of a triangle with sides 7, 24 and 25 cm in cm².	84 $s = 28$; $\sqrt{(28 \times 21 \times 4 \times 3)} = 84$.
04 Find the area of an isosceles triangle with sides 10, 10 and 12 cm in cm².	48 $s = 16$; $\sqrt{(16 \times 6 \times 6 \times 4)} = 48$.
05 A triangle has sides 8, 15 and 17 cm. Find its semiperimeter in cm.	20 The perimeter is 40, so $s = 20$.
06 Which side set forms an ordinary triangle? A) 0, 5, 5 B) 5, 6, 8 C) 2, 3, 5 D) 1, 4, 7	5, 6, 8 $5 + 6 > 8$, while the other choices are flat, impossible or have a zero side.

Recover the height from three sides

A triangle has sides 13, 14 and 15 cm. Its semiperimeter is 21 cm. Heron's product is $21 \times 8 \times 7 \times 6 = 7056$, so its area is 84 cm².

With 14 cm as base, height = $2 \times 84 \div 14 = 12$ cm. The sloping sides are longer than that perpendicular.



The hypotenuse is 17 and one leg is 8. Recover the other leg.

Subtract squares: 289 − 64 = 225. The missing positive length is 15.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 An equilateral triangle has side 4 cm. Which is its exact area? A) 8 cm² B) 16√3 cm² C) 12 cm² D) 4√3 cm²	4√3 cm² s = 6 gives √(6 × 2 × 2 × 2) = √48 = 4√3.
08 An equal-area square replaces a rectangle 16 cm by 9 cm. Find the square side in cm.	12 The area is 144 cm², so the positive square side is 12 cm.
09 How can you check a Heron result for a right triangle without using Heron again?	Identify the longest side and check the Pythagorean equality. Use the two perpendicular shorter sides as base and height. Calculate half their product and compare the area with the Heron result.
10 Why does a side set such as 4, 7, 11 not describe an ordinary triangle?	The two shorter sides sum exactly to the longest. They can only lie along one straight segment, giving a flattened configuration. The strict triangle inequality fails, so it is not a non-degenerate triangle.
11 A rectangle has area 225 cm². Find the side of an equal-area square in cm.	15 The positive square root of 225 is 15.

$$7^2 + 24^2$$
$$\downarrow$$
$$49 + 576 = 625$$
$$\downarrow$$
$$25^2$$

Can sides 7, 24 and 25 make a right triangle?

Yes. The converse places the right angle opposite the longest side, 25.

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