MATHS FIELD GUIDE · GRADE 9

Algebraic fractions and restrictions

OPENING PROBLEM

Where do the rows differ?

x	2	3	4
Original	5	undefined	7
Simplified	5	6	7

02 Record the forbidden values before cancelling
03 Rules and worked cases
04 From the givens to the answer
05 Split a rectangle to see the distributive law
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Record the forbidden values before cancelling

A denominator must stay nonzero in the original expression.

1

$$(x^2 - 9)/(x - 3)$$
$$\downarrow$$
$$(x - 3)(x + 3)/(x - 3)$$
$$\downarrow$$
$$x + 3 \text{ for } x \neq 3$$

Which input is still forbidden?

Three. The original expression has denominator zero there.

2

x	2	3	4
Original	5	undefined	7
Simplified	5	6	7

Where do the rows differ?

At 3 the simplified formula alone fills a hole that the original expression has.

3

$$(x + 2)/(x + 5)$$
$$\downarrow$$
$$x \neq -5$$

Can x be cancelled from this fraction?

No. It is part of two sums, not a common factor.

4

$$2x/(4x^2)$$
$$\downarrow$$
$$1/(2x)$$
$$\downarrow$$
$$x \neq 0$$

Check the cancellation at x = 2.

Both forms give 1/4. At zero neither is defined.

$(x - 2)/(x - 2)$
one where $x \neq 2$.

$0/(x - 2)$
zero where $x \neq 2$.

CHAPTER PROBLEM

Which value is excluded by the denominator x – 7?

7. The denominator is zero at x = 7.

MATHS REFERENCE · GRADE 9

Algebraic fractions and restrictions

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Record excluded inputs	$(x^2 - 9)/(x - 3) = x + 3$ for $x \neq 3$. The original division fails at 3.
Use a common denominator	$1/x + 1/(2x) = 3/(2x)$, with $x \neq 0$.
Cancel whole factors	$(2x + 2)/(x + 1) = 2$ for $x \neq -1$. Factoring the entire numerator makes cancellation valid.

Read the representations

An algebraic denominator must stay non-zero

An algebraic fraction is defined only where its denominator is non-zero.

For $(x^2 - 9)/(x - 3)$, the original expression excludes $x = 3$.

Write the restriction before cancelling; it belongs to the original fraction.

$$\frac{(x - 3)(x + 3)}{x - 3} = x + 3$$

Only when $x \neq 3$

The highlighted factor may be divided out because it is non-zero on the allowed domain.

Factoring reveals a common factor. Cancellation does not restore the excluded input.

Cancel factors, not separate terms

If a numerator is a product $A \cdot B$ and its denominator is A, then $AB/A = B$ wherever $A \neq 0$. Here A and B may themselves be expressions.

The cancellation divides numerator and denominator by the same non-zero factor.

You cannot delete an x-term from $(x + 2)/x$ because x is not a factor of the entire numerator.

Reversed subtraction introduces a negative factor

Compare $x - 4$ with $4 - x$: each is the negative of the other, so $4 - x = -(x - 4)$.

Factor the numerator first. The highlighted expressions match, but the multiplier -1 remains after cancellation.

At $x = 6$, the fraction is $-2/2 = -1$. Check the signs before cancelling.

$$4 - x = -1 \times (x - 4)$$

$$\frac{-1 \times (x - 4)}{x - 4} = -1$$

Keep $x \neq 4$

Divide out the highlighted factor. Keep its multiplier.

$$(x - 2)(x + 3) \downarrow x^2 + x - 6$$

Which inputs give zero?

2 and −3. Each makes a different factor vanish.

WORKED EXAMPLES

From the givens to the answer

Can x be cancelled from a sum?

A learner writes $(x + 5)/x = 5$.

1. The numerator is a sum, not a product containing x as a common factor.
2. For $x \neq 0$, splitting gives $1 + 5/x$.
3. At $x = 5$ the original is 2, so the proposed constant 5 cannot be equivalent.

Simplify $(t^2 + 5t + 6)/(t + 2)$

1. The denominator excludes $t = -2$.
2. Factor the numerator: $(t + 2)(t + 3)$.
3. For $t \neq -2$, cancel $t + 2$ to get $t + 3$.
4. Keep the restriction: $t + 3$ with $t \neq -2$.

Simplify $(2y^2 - 8)/(y + 2)$

1. The original denominator excludes $y = -2$.
2. Factor the numerator fully: $2(y - 2)(y + 2)$.
3. Cancel $y + 2$ for allowed inputs. The result is $2(y - 2)$, with $y \neq -2$.

Check another case

Example 1

Which is the correct simplification of $(q^2 - 9)/(q + 3)$?

$q - 3, q \neq -3$ · $q - 3, q \neq 3$ · $q + 3, q \neq -3$ · $q - 3$ with no restriction

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$q - 3, q \neq -3$

Cancel $q + 3$ only for $q \neq -3$.

Example 2

Explain the difference between cancelling a common factor and deleting a term.

A common factor multiplies the entire numerator and denominator.

Dividing both by the same non-zero factor preserves their ratio.

A term in a sum is not automatically such a factor.

Which table entry is the vertical intercept?

x	−3	0	2
p(x)	0	−6	0

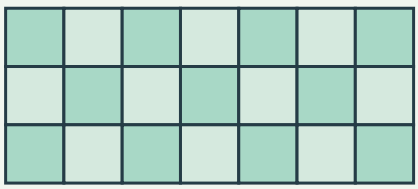
At $x = 0$ the output is -6. The two zero outputs locate horizontal intercepts.

ILLUSTRATED PROBLEM SHEET

Split a rectangle to see the distributive law

One common height multiplies every piece of the width.

1



Split this 3 by 7 array after five columns. What equality appears?

$3 \times 7 = 3 \times 5 + 3 \times 2$. The two pieces cover exactly the original rectangle.

2

$$4(x + 3)$$
$$\downarrow$$
$$4x + 4 \times 3$$
$$\downarrow$$
$$4x + 12$$

Which term is often missed?

The 3 is also multiplied by 4. At $x = 2$ both sides give 20.

3

$$5x + 15$$
$$\downarrow$$
$$5(x + 3)$$

Can you reverse the split?

Both terms contain 5. Dividing each by 5 leaves x and 3 inside the bracket.

4

$$-2(x - 4)$$
$$\downarrow$$
$$-2x + 8$$

Why is the constant positive?

It comes from multiplying two negative factors. Test $x = 0$: both sides give 8.

$2(x + y)$
$2x + 2y$.

$2x + y$
only x has coefficient 2.

CHAPTER PROBLEM

Find the excluded value for $(x + 1)/(x + 4)$.

-4. $x + 4 = 0$ gives $x = -4$.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Can 1/(x − 1) ever equal zero?	No. A nonzero numerator cannot give a zero fraction where its denominator exists.
02 Which correctly simplifies 6x/3 for all real x? A) 2x B) 2 C) 3x D) x/2	2x Cancel the non-zero numerical factor 3, giving 2x.
03 Is 3a + 2b always 5ab?	No. With a = b = 2 the two sides are 10 and 20. Unlike terms cannot be merged that way.
04 For x ≠ 5, what does (x² − 25)/(x − 5) simplify to? A) x − 5 B) x + 5 C) x² + 5 D) 1	x + 5 Factor as (x − 5)(x + 5), then cancel the non-zero common factor.
05 Which restriction remains after simplifying (p² − 16)/(p + 4)? A) p ≠ 4 B) p ≠ −4 C) p > 4 D) No restriction	p ≠ −4 The original denominator vanishes at −4.
06 Evaluate (x² − 1)/(x − 1) at x = 6.	7 At the allowed input 6, the simplified expression x + 1 gives 7.

2(x − 2)(x + 3)
↓
2x² + 2x − 12

Does doubling the polynomial change its roots?

No. A nonzero constant factor changes the outputs but preserves the zeros.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Which cancellation is valid for a ≠ 0? A) (a + 3)/a = 3 B) (a − 3)/a = −3 C) 3a/a = 3 D) (a² + 1)/a = a + 1	3a/a = 3 a is a factor of the whole numerator 3a.
08 For x ≠ 0, evaluate (2x + 6)/x at x = 3.	4 (6 + 6)/3 = 4.
09 Why must x ≠ 2 remain when (x² − 4)/(x − 2) simplifies to x + 2?	The original denominator equals zero at 2. Cancellation is valid only when x − 2 is non-zero. The two rules describe the same expression only when that exclusion is retained.
10 Explain why (x + 1)/x is not equal to 1 for every non-zero x.	x is not a factor of the entire numerator. The expression equals 1 + 1/x. For example x = 1 gives 2, disproving the claim.
11 Which value is excluded by denominator 2x + 6?	-3 2x + 6 = 0 gives x = −3.

Sum = −1
↓
Product = −6
↓
x² + x − 6

Reconstruct a monic quadratic from these two root facts.

Use x² − (sum)x + product. Its factors are (x − 2) and (x + 3).

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