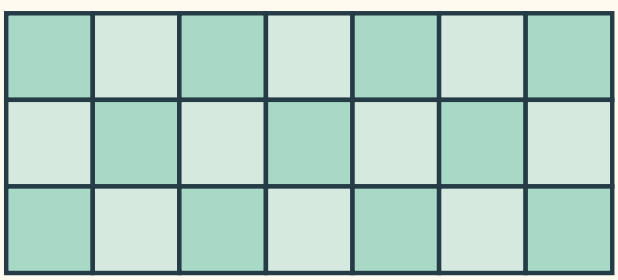
MATHS FIELD GUIDE · GRADE 9

Algebraic expressions and factor structure

OPENING PROBLEM

Split this 3 by 7 array after five columns. What equality appears?



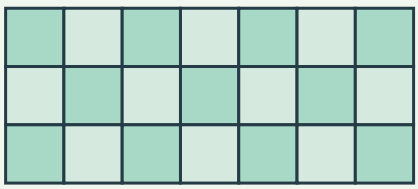
02 Split a rectangle to see the distributive law
03 Rules and worked cases
04 From the givens to the answer
05 Match each pattern to its expansion
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Split a rectangle to see the distributive law

One common height multiplies every piece of the width.

1



Split this 3 by 7 array after five columns. What equality appears?

$3 \times 7 = 3 \times 5 + 3 \times 2$. The two pieces cover exactly the original rectangle.

2

$$4(x + 3)$$
$$\downarrow$$
$$4x + 4 \times 3$$
$$\downarrow$$
$$4x + 12$$

Which term is often missed?

The 3 is also multiplied by 4. At $x = 2$ both sides give 20.

3

$$5x + 15$$
$$\downarrow$$
$$5(x + 3)$$

Can you reverse the split?

Both terms contain 5. Dividing each by 5 leaves x and 3 inside the bracket.

4

$$-2(x - 4)$$
$$\downarrow$$
$$-2x + 8$$

Why is the constant positive?

It comes from multiplying two negative factors. Test $x = 0$: both sides give 8.

$2(x + y)$
$2x + 2y$.

$2x + y$
only x has coefficient 2.

CHAPTER PROBLEM

Evaluate $2(x + 3)$ when $x = 4$.

14. $2 \times (4 + 3) = 14$.

MATHS REFERENCE · GRADE 9

Algebraic expressions and factor structure

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Combine like terms	$3x^2 + 4x - x^2 = 2x^2 + 4x$. Exponents determine which terms match.
Extract a factor	$6x^2 - 9x = 3x(2x - 3)$. Distributing reproduces both original terms.
Group terms	$ax + ay + bx + by = a(x + y) + b(x + y) = (a + b)(x + y)$.

Read the representations

An expression describes a value

A variable stands for a number. In $5x + 2$, x may change, while 5 and 2 are constants.

A term is a part separated by addition or subtraction. The coefficient of x in $5x$ is 5.

An expression has a value once its variables are specified. An equation makes a claim that two values are equal.

$5x + 2$

5

Coefficient of x

x

Variable

2

Constant term

At $x = 3$, the expression has value 17.

The coefficient multiplies the variable. The constant term stands alone.

Like terms count the same kind of quantity

$3x + 4x = 7x$ because both terms count copies of x.

$3x + 4$ cannot be collected into one x-term. Likewise, x and x^2 are different kinds of term.

Include the sign with a term: $6x - 2x$ has coefficients 6 and -2. The table keeps each variable-and-power family separate.

Only terms in the same row combine

Family	Collect coefficients	Result
x terms	$3x + 4x$	$7x$
x^2 terms	$2x^2 + 5x^2$	$7x^2$
Constants	$2 + 6$	8

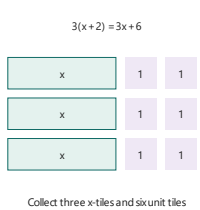
For example, $x^2 + x + 1$ stays as three unlike terms.

Expansion and factorisation reverse each other

In $3(x + 2)$, the multiplier 3 applies to both terms. The result is $3x + 6$.

Reading backwards, 3 is a common factor of $3x$ and 6, so $3x + 6 = 3(x + 2)$.

Expand a proposed factorisation to check that it returns the original expression.



Three rows each contain one x-tile and two unit tiles. The tile lengths represent positive x.

$(x - 2)(x + 3)$

↓

$x^2 + x - 6$

Which inputs give zero?

2 and -3. Each makes a different factor vanish.

WORKED EXAMPLES

From the givens to the answer

Does 2(x + 5) equal 2x + 5?

A learner multiplies the x but leaves the 5 unchanged.

1. Each of the two groups contains an x and five ones.
2. The five ones occur twice too, giving 2x + 10.
3. Check at x = 1: the original is 12, while 2x + 5 would be 7.

Simplify 4(2t – 3) + 5t

1. Distribute 4: 8t – 12 + 5t.
2. Collect only the t-terms: 8t + 5t = 13t.
3. The result is 13t – 12. Check at t = 2: both expressions give 14.

Factor 12y + 18 fully

1. The greatest common numerical factor is 6.
2. 12y = 6 × 2y and 18 = 6 × 3.
3. Therefore 12y + 18 = 6(2y + 3). Expanding checks the result.

Check another case

Example 1

Which fully factors 15x – 20?

5(3x – 4) · 5(3x – 20) · 15(x – 4) · x(15 – 20)

5(3x – 4)

5 is common to both terms; expanding gives 15x – 20.

Example 2

Explain why 2x + 3x² contains two unlike terms.

One variable part is x and the other is x².

They represent different powers of the same variable.

Their coefficients cannot be added to form a single like term.

Which table entry is the vertical intercept?

x	–3	0	2
p(x)	0	–6	0

At x = 0 the output is –6. The two zero outputs locate horizontal intercepts.

ILLUSTRATED PROBLEM SHEET

Match each pattern to its expansion

Multiplying two sums forms every pairwise product.

1

×	a	b
a	a^2	ab
b	ab	b^2

Where does 2ab in $(a + b)^2$ come from?

The two rectangular off-diagonal products are both ab.

2

$$(3x - 2)^2$$
$$\downarrow$$
$$(3x)^2 - 2(3x)(2) + 2^2$$
$$\downarrow$$
$$9x^2 - 12x + 4$$

Which signs change in a squared difference?

The middle product is negative. The constant is positive because $(-2)^2 = 4$.

3

$$49x^2 - 16$$
$$\downarrow$$
$$(7x)^2 - 4^2$$
$$\downarrow$$
$$(7x - 4)(7x + 4)$$

Why is there no middle term?

The cross-products +28x and −28x cancel.

4

×	x	−3
x	x^2	−3x
5	5x	−15

Collect the products of $(x + 5)(x - 3)$.

The middle terms combine to 2x, giving $x^2 + 2x - 15$.

$(p + q)^2 - (p - q)^2$
4pq.

$(p + q)^2 + (p - q)^2$
$2p^2 + 2q^2$.

CHAPTER PROBLEM

Find the coefficient of x in $9x - 4x + 6$.

5. The x-coefficient is 9 − 4 = 5.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Which terms can be collected? A) 3a and 2a B) 3a and 2 C) a and a^2 D) a and b	3a and 2a Both 3a and 2a are multiples of a.
02 Is 3a + 2b always 5ab?	No. With a = b = 2 the two sides are 10 and 20. Unlike terms cannot be merged that way.
03 Which expression equals 5(x − 2)? A) 5x − 2 B) 5x − 10 C) x − 10 D) 3x	5x − 10 Distribution gives 5x − 10.
04 Which is 7p − 3p + 2 simplified? A) 4p + 2 B) 6p C) 10p + 2 D) $4p^2 + 2$	4p + 2 7p − 3p = 4p; the constant 2 stays separate.
05 Use one identity to calculate 99^2 without long multiplication.	$(100 - 1)^2 = 10{,}000 - 200 + 1 = 9801$.
06 Evaluate 3z − 5 when z = −2.	-11 3(−2) − 5 = −6 − 5 = −11.

Which expression is always even?

For an integer n, compare $n^2 + n$ and $n^2 + 1$. Factor the first as n(n + 1). Of two consecutive integers, one is even, so their product is even.

The second expression fails at n = 2, where it is 5. One counterexample settles an “always” claim.

$$2(x - 2)(x + 3) \downarrow 2x^2 + 2x - 12$$

Does doubling the polynomial change its roots?

No. A nonzero constant factor changes the outputs but preserves the zeros.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Which is a fully factored form of 8a + 12? A) 2(4a + 6) B) 4(2a + 3) C) 8(a + 12) D) 4(2a + 12)	4(2a + 3) 4 is the greatest common factor and the bracket has no further common factor.
08 Evaluate 2(3t + 1) − t when t = 3.	17 2(10) − 3 = 17, or simplify to 5t + 2 first.
09 Explain why 4x + 3 cannot be simplified to 7x.	4x counts four copies of x, while 3 is a constant. The terms have different variable parts. For example x = 2 gives 11, whereas 7x gives 14.
10 Explain how expansion checks 7(2r − 1) as a factorisation of 14r − 7.	Multiply 7 by 2r to obtain 14r. Multiply 7 by −1 to obtain −7. The expansion exactly matches both original terms.
11 Evaluate 6(x − 1) when x = 3.	12 6 × 2 = 12.

Sum = −1
↓
Product = −6
↓
$x^2 + x - 6$

Reconstruct a monic quadratic from these two root facts.

Use x^2 − (sum)x + product. Its factors are (x − 2) and (x + 3).

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