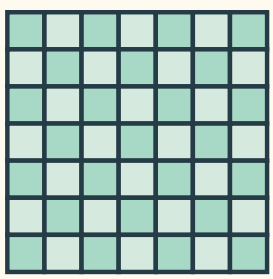
MATHS FIELD GUIDE · GRADE 8

Roots undo powers

OPENING PROBLEM

Which square has area 49 square units?



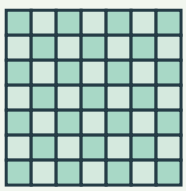
02 Bound a root and test the claim
03 Rules and worked cases
04 Find the side from the area
05 Count the factors before using a power rule
06 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Bound a root and test the claim

Squaring a nonnegative number preserves order. The radical sign takes the nonnegative root.

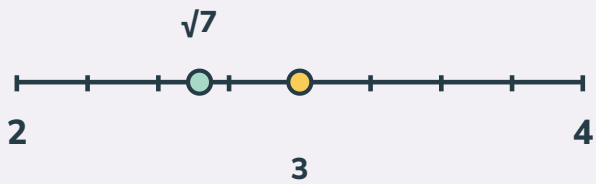
1



Which square has area 49 square units?

Seven rows of seven. Its side length is $\sqrt{49} = 7$.

2



Which integers trap √7?

$2^2 < 7 < 3^2$. Hence $2 < \sqrt{7} < 3$.

3

$$3.3^2 = 10.89$$
$$\downarrow$$
$$11$$
$$\downarrow$$
$$3.4^2 = 11.56$$

Which tenths trap √11?

$3.3 < \sqrt{11} < 3.4$. Compare squares before rounding the root.

4

$$x^2 = 25$$
$$\downarrow$$
$$x = 5 \text{ or } x = -5$$
$$\downarrow$$
$$\sqrt{25} = 5$$

Why are there two equation solutions but one square root symbol value?

Both signs square to 25. The radical denotes the nonnegative one.

$\sqrt{81}$
positive 9.

$x^2 = 81$
two real solutions, 9 and −9.

CHAPTER PROBLEM

Explain the difference between $\sqrt{36}$ and the solutions to $x^2 = 36$.

$\sqrt{36}$ is 6. The equation has solutions 6 and −6 because both square to 36.

MATHS REFERENCE · GRADE 8

Roots undo powers

A square root of a non-negative number is a value whose square is that number. The symbol $\sqrt{a}$ denotes the non-negative, principal square root.

An irrational number cannot be written as a ratio of two integers. Its decimal representation does not terminate or repeat.

Four cases to recognise

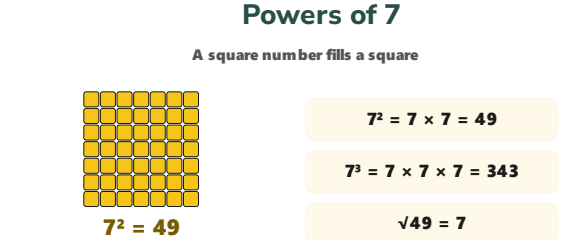
Situation	What to do and why
$\sqrt{0.49}$	$0.7^2 = 0.49$, so the principal square root is 0.7.
$\sqrt[3]{(-125)}$	$(-5)^3 = -125$, so the cube root is −5.
Bracket $\sqrt{70}$	$8^2 = 64$ and $9^2 = 81$, so $8 < \sqrt{70} < 9$.
Solve $x^2 = 0.81$	Both +0.9 and −0.9 square to 0.81. The radical $\sqrt{0.81}$ denotes +0.9.

Read the model

Build the idea

A square with area 49 has side length seven, because $7 \times 7 = 49$. Thus $\sqrt{49} = 7$.

For area 50, the side length is just above seven and below eight because $49 < 50 < 64$.

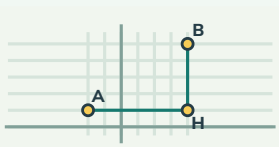


Why it works

Squaring and taking a principal square root reverse one another for non-negative values. The equation $x^2 = 49$ has two real solutions, +7 and −7.

The notation $\sqrt{49}$ refers only to +7. Distinguish the value of a square-root expression from all solutions to an equation.

principal root The non-negative square root.
irrational Not expressible as an integer ratio.
exact form A value written without rounding.



A (-2, 1)
H (4, 1)
B (4, 5)

Find the direct distance from A(−2, 1) to B(4, 5).

The legs are 6 and 4. AB = $\sqrt{52} = 2\sqrt{13}$. The broken route via H is 10 units.

WORKED INVESTIGATION

Find the side from the area

A square tile covers 196 cm². Its side is the positive number whose square is 196. Use nearby square numbers to bracket it.

Look at this part	What it tells you
13^2	169
14^2	196
15^2	225

Work it through

Area 200 cm² gives a side between 14 and 15 cm. Testing $14.1^2 = 198.81$ and $14.2^2 = 201.64$ narrows the interval.

Catch the mistake

The equation $x^2 = 196$ has answers 14 and −14. A physical length uses the positive root only.

Compare the cases

Follow an example

$\sqrt{50} = \sqrt{(25 \times 2)} = 5\sqrt{2}$. This is an exact form; a decimal approximation is about 7.071.

A number-line estimate locates $\sqrt{50}$ between 7 and 8. Better bounds can be tested by squaring decimal estimates.

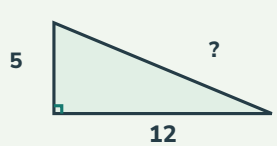
What changes? What stays true?

In real-number work, a negative number has no real square root. The square of any real number is non-negative.

Square roots do not distribute over addition: $\sqrt{(9 + 16)} = 5$, whereas $\sqrt{9} + \sqrt{16} = 7$.

Change one condition

For cube volume, undo the third power instead: a cube of volume 216 cm³ has edge 6 cm since $6^3 = 216$.



A right triangle has legs 5 and 12. How long is its hypotenuse?

$5^2 + 12^2 = 169$, so the longest side is 13.

ILLUSTRATED PROBLEM SHEET

Count the factors before using a power rule

Multiplying equal bases joins factor lists. Dividing cancels matching factors when the base is nonzero.

1

$$2^3 \times 2^2$$
$$\downarrow$$
$$(2 \times 2 \times 2) \times (2 \times 2)$$
$$\downarrow$$
$$2^5 = 32$$

Why add the exponents?

All five factors have the same base.

2

$$3^4 \div 3^2$$
$$\downarrow$$
$$(3 \times 3 \times 3 \times 3) \div 9$$
$$\downarrow$$
$$3^2 = 9$$

Why subtract the exponents?

Two numerator factors cancel with two denominator factors.

3

$$5^0 = 5^3 \div 5^3$$
$$\downarrow$$
$$125 \div 125$$
$$\downarrow$$
$$1$$

What condition matters for a zero exponent?

Use a nonzero base so that the division exists.

4

$$0.0048$$
$$\downarrow$$
$$4.8 \times 10^{-3}$$
$$\downarrow$$
$$4.8 \div 1000$$

How does the minus sign change the scale?

It selects the reciprocal power of ten. The number itself stays positive.

$10^{-2} \times 10^5$
10^3.

$10^{-2} \div 10^5$
10^{-7}.

CHAPTER PROBLEM

Between which consecutive integers does $\sqrt{70}$ lie? Give evidence without rounding.

$8^2 = 64$ and $9^2 = 81$. Since $64 < 70 < 81$, the positive square root lies between 8 and 9.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

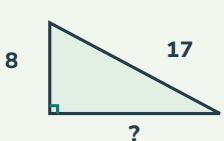
Check your reasoning

Try the problem	Solution and reasoning
01 Is $\sqrt{(9 + 16)}$ equal to $\sqrt{9} + \sqrt{16}$?	No. The first is 5. The second is 3 + 4 = 7.
02 Is $2^3 + 2^2$ equal to 2^5?	No. Addition combines the values 8 and 4 to give 12. The product rule applies to multiplication.
03 Is the side of a square of area 150 cm² closer to 12 or 13 cm? Justify.	It is between 12 and 13, and $12.5^2 = 156.25$ exceeds 150. The side is below the midpoint, so it is closer to 12 cm.
04 Bracket $\sqrt[3]{30}$ between two integers.	$3^3 = 27$ and $4^3 = 64$, so $3 < \sqrt[3]{30} < 4$.
05 Can a real square root of −16 exist?	No real number has a negative square.

Between which tenths is √11?

First compare $3^2 = 9$ and $4^2 = 16$. Next, $3.3^2 = 10.89$ and $3.4^2 = 11.56$. These trap $\sqrt{11}$ between 3.3 and 3.4.

The positive square root increases as the positive number increases. This gives a bound without rounding $\sqrt{11}$ first.



The hypotenuse is 17 and one leg is 8. Recover the other leg.

Subtract squares: 289 − 64 = 225. The missing positive length is 15.

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