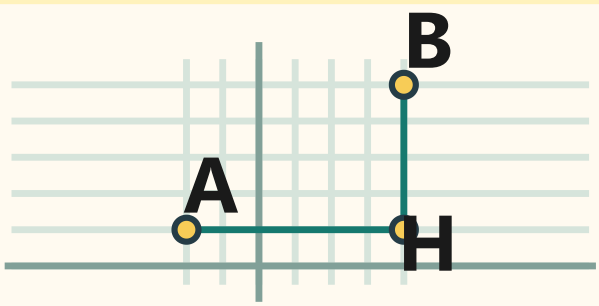
MATHS FIELD GUIDE · GRADE 8

Pythagoras relates square areas

OPENING PROBLEM

Find the direct distance from A(−2, 1) to B(4, 5).



A (-2, 1)
H (4, 1)
B (4, 5)

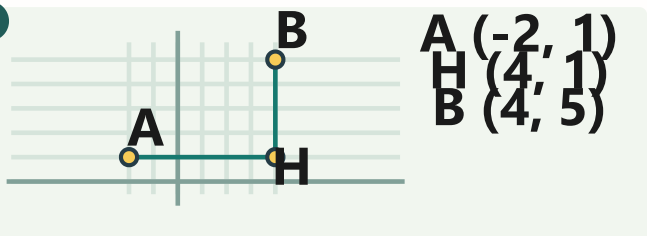
02 Build the right triangle between two points
03 Rules and worked cases
04 Can the ladder reach?
05 Construct a length from perpendicular steps
06 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Build the right triangle between two points

The horizontal and vertical displacements are perpendicular. Their squares add to the squared direct distance.

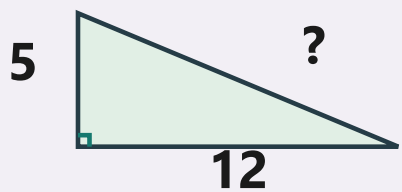
1



Find the direct distance from A(−2, 1) to B(4, 5).

The legs are 6 and 4. AB = $\sqrt{52} = 2\sqrt{13}$. The broken route via H is 10 units.

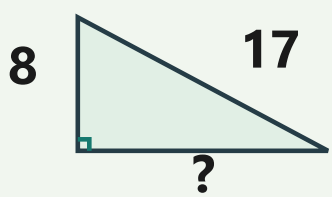
2



A right triangle has legs 5 and 12. How long is its hypotenuse?

$5^2 + 12^2 = 169$, so the longest side is 13.

3



The hypotenuse is 17 and one leg is 8. Recover the other leg.

Subtract squares: 289 − 64 = 225. The missing positive length is 15.

4

$$7^2 + 24^2$$
$$\downarrow$$
$$49 + 576 = 625$$
$$\downarrow$$
$$25^2$$

Can sides 7, 24 and 25 make a right triangle?

Yes. The converse places the right angle opposite the longest side, 25.

Coordinate changes (0,7)
distance 7.

Coordinate changes (−3,4)
distance 5.

CHAPTER PROBLEM

A triangle has side lengths 5, 12 and 13. How can you justify that it is right-angled?

Its longest side is 13, and $5^2 + 12^2 = 13^2$. The converse of Pythagoras establishes a right angle.

MATHS REFERENCE · GRADE 8

Pythagoras relates square areas

In a right triangle, the square on the hypotenuse has the same area as the two squares on the other sides together.

The hypotenuse lies opposite the right angle. If the shorter side lengths are a and b and the hypotenuse length is c, the relationship is $a^2 + b^2 = c^2$.

Four cases to recognise

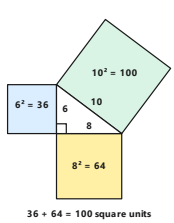
Situation	What to do and why
Legs 7 and 24	Hypotenuse squared is 49 + 576 = 625. Length 25.
Hypotenuse 17 and one leg 8	Other leg $\sqrt{(289 - 64)} = 15$.
Lengths 6, 8 and 11	36 + 64 differs from 121. The triangle is not right-angled.
Diagonal of a square of side 5	$d^2 = 25 + 25 = 50$, so $d = 5\sqrt{2}$.

Read the model

Build the idea

Draw a right triangle with legs 6 and 8. Build a square on each side. The smaller squares have areas 36 and 64.

Their combined area is 100. A square of area 100 has side length 10, giving the hypotenuse. The formula concerns areas before it determines a length.

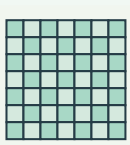


Why it works

For a proof, arrange four copies of a right triangle inside a square of side a + b. The uncovered central square has side c.

The outer area is $(a + b)^2$ and the four triangles total 2ab. Subtracting leaves $a^2 + b^2$, which also equals c^2. This explains why the relationship holds for every right triangle.

hypotenuse Side opposite a right triangle's right angle.
theorem A statement established by proof.
converse A statement formed by reversing a theorem's condition and conclusion.



Which square has area 49 square units?

Seven rows of seven. Its side length is $\sqrt{49} = 7$.

WORKED INVESTIGATION

Can the ladder reach?

A ladder rests against a vertical wall on level ground. Its foot is 5 m from the wall and its top is 12 m high. The ladder is opposite the right angle.

Look at this part	What it tells you
Ground segment squared	25 m²
Wall segment squared	144 m²
Ladder squared	169 m²

Work it through

The ladder length is $\sqrt{169} = 13$ m. It exceeds either leg but is shorter than their sum, as the triangle requires.

Catch the mistake

When the ladder length is given, subtract a leg's square to find the other leg's square. Adding squares would produce a length longer than the ladder.

Compare the cases

Follow an example

For legs 5 and 12, $c^2 = 25 + 144 = 169$, so $c = 13$. Take the non-negative root because a length cannot be negative.

If $c = 13$ and one leg is 5, the other leg satisfies $b^2 = 169 - 25 = 144$, giving $b = 12$.

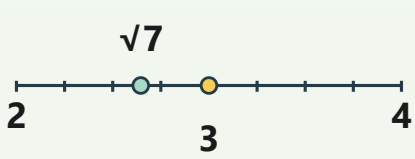
What changes? What stays true?

Check for a right angle before using the theorem. A triangle's appearance or its longest-looking side is insufficient evidence.

The converse is also useful: if the square of the longest side equals the sum of the other side squares, the triangle is right-angled. Area equality must use the same units throughout.

Change one condition

With the same 13 m ladder, moving the foot to 12 m from the wall lowers the height to 5 m. The two legs exchange roles.



Which integers trap √7?

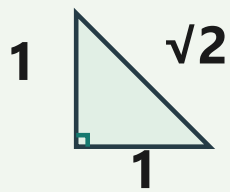
$2^2 < 7 < 3^2$. Hence $2 < \sqrt{7} < 3$.

ILLUSTRATED PROBLEM SHEET

Construct a length from perpendicular steps

A new perpendicular leg gives a right triangle. Label the square of the new radius before taking the root.

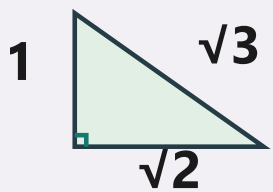
1



From a unit segment, add another unit at right angles. What is the diagonal?

Its square is 1 + 1 = 2, so its positive length is √2.

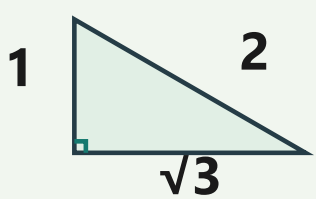
2



Build a unit leg perpendicular to the new radius. What comes next?

The squared radius becomes 2 + 1 = 3.

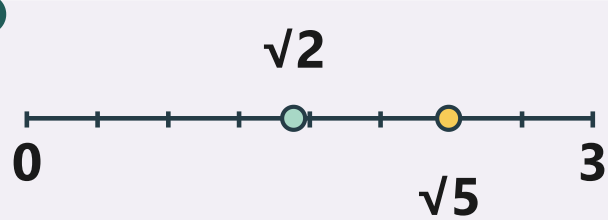
3



What happens on the next unit step?

The squared length is four. The radius is now the integer two.

4



How can a length become a point on a number line?

Set a compass to the constructed length. With centre zero, transfer it to the positive ray.

Radius squared 7, new perpendicular unit step
squared radius 8.

The length increase
√8 − √7, less than one.

CHAPTER PROBLEM

A 13 m ladder reaches a point 12 m up a vertical wall. How far is its foot from the wall on level ground?

The ladder is the hypotenuse. The horizontal distance is $\sqrt{(13^2 - 12^2)} = \sqrt{25} = 5$ m.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Is a route 3 units east and 4 north 7 units from its start?	The route length is 7. The straight-line distance is 5.
02 Could the added unit segment point in an arbitrary direction and give the same calculation?	No. The perpendicular condition is what allows addition of squares.
03 A 10 m ladder has its foot 6 m from a vertical wall. Find the height reached.	The height squared is 100 − 36 = 64. The positive square root gives 8 m.
04 A rectangle is 9 by 12. What is the length of its diagonal?	√(81 + 144) = 15.
05 Two legs are both 6. Is the hypotenuse 12?	No. Its length is √72 = 6√2, less than the sum of the two legs.

$3.3^2 = 10.89$
↓
11
↓
$3.4^2 = 11.56$

Which tenths trap √11?

3.3 < √11 < 3.4. Compare squares before rounding the root.

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