MATHS FIELD GUIDE · GRADE 8

Solving equations preserves their solution sets

OPENING PROBLEM

Which change keeps both sides balanced?

$$x + 7 = 19$$
$$\downarrow$$
$$x = 19 - 7$$
$$\downarrow$$
$$x = 12$$

02 Keep the solution set intact
03 Rules and worked cases
04 Compare two taxi fares
05 Decide what the intersection means
06 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Keep the solution set intact

An operation used on both sides must be reversible on the allowed values.

1

$$x + 7 = 19$$
$$\downarrow$$
$$x = 19 - 7$$
$$\downarrow$$
$$x = 12$$

Which change keeps both sides balanced?

Subtract seven from each side. Substitution gives 12 + 7 = 19.

2

$$3x - 4 = 14$$
$$\downarrow$$
$$3x = 18$$
$$\downarrow$$
$$x = 6$$

Which step undoes multiplication?

Dividing both sides by three. Check: 18 – 4 = 14.

3

$$2(x + 3) = 2x + 6$$
$$\downarrow$$
$$2x + 6 = 2x + 6$$
$$\downarrow$$
$$0 = 0$$

What does this final equality mean?

Every allowed x satisfies the original equation.

4

$$2(x + 3) = 2x + 7$$
$$\downarrow$$
$$6 = 7$$

What if the variable disappears into a false statement?

There is no solution. Subtracting the same 2x did not lose one.

0x = 0
all real x fit.

0x = 4
no real x fits.

CHAPTER PROBLEM

What does it mean if solving an equation leaves 0 = 5?

It is a contradiction. The original equation has no solution in the stated domain.

MATHS REFERENCE · GRADE 8

Solving equations preserves their solution sets

A linear equation uses the variable to the first power, without products of the variable with itself. Solving seeks all values that make its two expressions equal.

Equivalent equations have the same solution set. A valid rearrangement must preserve that set.

Four cases to recognise

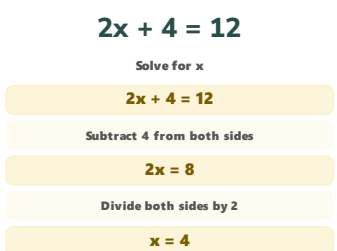
Situation	What to do and why
4(x − 2) = 2x + 6	Expand and collect: 2x = 14, so x = 7.
3(x + 1) = 3x + 3	Both sides expand identically. Every real x works.
2(x − 1) = 2x + 1	Cancelling 2x gives −2 = 1. No solution.
Half a number plus three is eleven	x/2 + 3 = 11 gives x = 16. Check eight plus three.

Read the model

Build the idea

For 3x + 4 = x + 12, remove x from both sides to obtain 2x + 4 = 12. Remove four from both sides, giving 2x = 8.

Divide both sides by two, obtaining x = 4. The unknown can appear on both sides without changing the balance principle.



Solving a linear equation

Why it works

Each step changes equal quantities in the same reversible way. This is why the final value must satisfy the original equation.

Some equations simplify to a true identity, while others simplify to a contradiction. The number of solutions depends on the relationship.

linear equation An equation with first-power variable terms.
identity Equality holding for all allowed values.
contradiction A statement that cannot hold.

? + 18 = 43
↓
43 − 18 = 25

What number was hidden?

Twenty-five. Adding eighteen to it returns forty-three.

WORKED INVESTIGATION

Compare two taxi fares

Taxi A charges ₹40 plus ₹8 per kilometre. Taxi B charges ₹20 plus ₹12 per kilometre. Equal fares mean 40 + 8d = 20 + 12d.

Look at this part	What it tells you
At 0 km	A ₹40, B ₹20
At 5 km	both ₹80
At 8 km	A ₹104, B ₹116

Work it through

Subtract 8d and then 20 from both sides. The remaining equation 20 = 4d says the higher starting price is offset by ₹4 per kilometre.

Catch the mistake

An equality at 5 km does not make the two fare rules identical. Their rates differ, so each extra kilometre separates them again.

Compare the cases

Follow an example

2(x + 3) = 2x + 6 expands to 2x + 6 = 2x + 6. Every real x satisfies it.

2(x + 3) = 2x + 7 instead gives 6 = 7 after subtracting 2x. No value of x can make it true.

What changes? What stays true?

Do not divide by an expression that might be zero without considering that case. A lost case can change the solution set.

A modelled answer also needs a context check: an equation may allow a negative length algebraically, but the physical problem may not.

Change one condition

If taxi A also charges ₹12 per kilometre, both fares rise at the same rate. The ₹20 gap persists at every distance.

? − 16 = 27
↓
27 + 16 = 43

A box lost sixteen and now has twenty-seven. How many at first?

Forty-three. Undo a loss by putting it back.

ILLUSTRATED PROBLEM SHEET

Decide what the intersection means

Each equation selects a line. A shared point satisfies both equations.

1

x	0	1	2
y = x + 1	1	2	3
y = 5 − x	5	4	3

Where do the lines meet?

At (2, 3). Both output rows give 3 in the same column.

2

$$y = x + 1$$
$$\downarrow$$
$$x + y = 5$$
$$\downarrow$$
$$x + (x + 1) = 5$$

Substitute to recover the same pair.

2x = 4, so x = 2. Then y = 3.

3

$$x + y = 5$$
$$\downarrow$$
$$-x + y = 1$$
$$\downarrow$$
$$2y = 6$$

Why add the equations?

The x terms cancel. y = 3, then x = 2.

4

Line A	Line B	Solutions
y = 2x + 1	y = 2x + 4	none
y = 2x + 1	2y = 4x + 2	infinite

What changes when the slopes match?

Different intercepts give parallel lines. Matching intercepts give the very same line.

Same slope, different intercepts
no solution.

Different slopes
exactly one solution.

CHAPTER PROBLEM

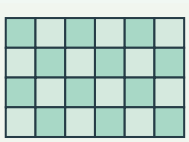
Solve 5x − 2 = 3x + 8. Check both sides at your solution.

Subtract 3x and add 2 on both sides: 2x = 10, so x = 5. Both sides then equal 23.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Why can dividing $x^2 = 3x$ by x miss an answer?	It assumes x is nonzero. Factoring $x(x - 3) = 0$ keeps both 0 and 3.
02 A notebook and pen cost ₹40. Two notebooks plus that pen cost ₹65. Find each price.	Subtract the totals: notebook ₹25. The pen is ₹15. Check both purchases.
03 Taxi C charges ₹10 plus ₹14 per kilometre. At what distance does C cost the same as A?	Solve $10 + 14d = 40 + 8d$. Then $6d = 30$ and $d = 5$ km. Both cost ₹80.
04 Solve $3(x - 2) = x + 10$.	$3x - 6 = x + 10$, so $2x = 16$ and $x = 8$.
05 If $5x + 1 = 5x + k$ has every real x as a solution, what is k?	Cancel 5x. The remaining equality $1 = k$ must always hold, so $k = 1$.



Twenty-four counters fill four identical rows. How many per row?

Six. Four times six gives the original total.

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