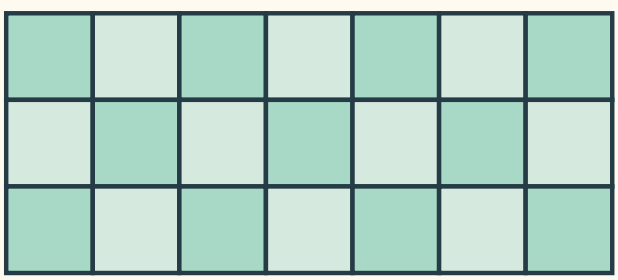
MATHS FIELD GUIDE · GRADE 8

Expansion and factorisation are inverse descriptions

OPENING PROBLEM

Split this 3 by 7 array after five columns. What equality appears?



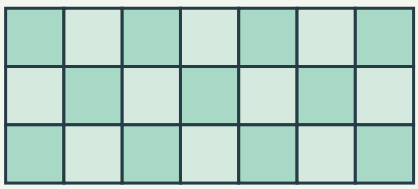
02 Split a rectangle to see the distributive law
03 Rules and worked cases
04 Two descriptions of a tiled floor
05 Match each pattern to its expansion
06 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Split a rectangle to see the distributive law

One common height multiplies every piece of the width.

1



Split this 3 by 7 array after five columns. What equality appears?

$3 \times 7 = 3 \times 5 + 3 \times 2$. The two pieces cover exactly the original rectangle.

2

$$4(x + 3)$$
$$\downarrow$$
$$4x + 4 \times 3$$
$$\downarrow$$
$$4x + 12$$

Which term is often missed?

The 3 is also multiplied by 4. At $x = 2$ both sides give 20.

3

$$5x + 15$$
$$\downarrow$$
$$5(x + 3)$$

Can you reverse the split?

Both terms contain 5. Dividing each by 5 leaves x and 3 inside the bracket.

4

$$-2(x - 4)$$
$$\downarrow$$
$$-2x + 8$$

Why is the constant positive?

It comes from multiplying two negative factors. Test $x = 0$: both sides give 8.

$2(x + y)$
$2x + 2y$.

$2x + y$
only x has coefficient 2.

CHAPTER PROBLEM

How can you verify that $5(2x + 3)$ is a factorisation of $10x + 15$?

Expand the product: five times 2x and five times 3 give $10x + 15$ for every x.

MATHS REFERENCE · GRADE 8

Expansion and factorisation are inverse descriptions

Expansion writes a product as a sum of parts. Factorisation writes a sum as a product. Equivalent forms describe the same quantity for every allowed input.

A common factor belongs to each term and can be taken outside a bracket.

Four cases to recognise

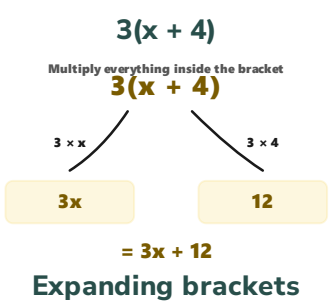
Situation	What to do and why
2(x + 5) − (x − 3)	Distribute both signs: 2x + 10 − x + 3 = x + 13.
(x + 3)(x − 3)	The middle terms cancel. Product $x^2 - 9$.
Factor $4x^2 - 12x$	The common factor is 4x, giving 4x(x − 3).
Do $x^2 + x$ and 2x agree always?	They match at x = 1 but differ at x = 2: six versus four.

Read the model

Build the idea

A rectangle with one side 3 and the other x + 4 has area 3(x + 4). Split it into widths x and 4.

The two areas are 3x and 12, so 3(x + 4) = 3x + 12. Regrouping the two rectangles reverses this expansion.



Expanding brackets

Why it works

The distributive law follows from the whole area equalling the sum of non-overlapping parts. The same identity applies beyond literal positive side-length models.

For 6x + 9, both terms have factor three. Taking it out leaves 3(2x + 3).

expand Replace a product by its distributed sum.
factorise Rewrite as a product.
identity Equality true for every permitted variable value.

x + 7 = 19
↓
x = 19 − 7
↓
x = 12

Which change keeps both sides balanced?

Subtract seven from each side. Substitution gives 12 + 7 = 19.

WORKED INVESTIGATION

Two descriptions of a tiled floor

A rectangular floor measures x + 2 by x + 3 units. Split both dimensions at x and calculate the four partial areas.

Look at this part	What it tells you
Large square	x^2
Two strips	2x + 3x
Corner	6

Work it through

Adding the pieces gives $x^2 + 5x + 6$. At x = 4 the rectangle is 6 by 7 and both expressions give 42 square units.

Catch the mistake

Expanding as $x^2 + 6$ loses both rectangular strips. A numerical check at x = 1 gives an immediate counterexample.

Compare the cases

Follow an example

Expand 2(3x − 5): two copies of 3x give 6x, and two copies of −5 give −10. The result is 6x − 10.

Factorise 8x + 12 as 4(2x + 3). Multiply back out to verify the factorisation.

What changes? What stays true?

A picture with lengths is a useful model for positive dimensions; algebra extends the relationship to negative values too.

Do not confuse factorising an expression with solving an equation. No unknown value is determined until a condition is stated.

Change one condition

To factor $x^2 + 5x + 6$, seek two numbers with product six and sum five. Two and three reconstruct the side lengths.

$$3x - 4 = 14$$
$$\downarrow$$
$$3x = 18$$
$$\downarrow$$
$$x = 6$$

Which step undoes multiplication?

Dividing both sides by three. Check: 18 − 4 = 14.

ILLUSTRATED PROBLEM SHEET

Match each pattern to its expansion

Multiplying two sums forms every pairwise product.

1

×	a	b
a	a^2	ab
b	ab	b^2

Where does 2ab in $(a + b)^2$ come from?

The two rectangular off-diagonal products are both ab.

2

$$(3x - 2)^2$$
$$\downarrow$$
$$(3x)^2 - 2(3x)(2) + 2^2$$
$$\downarrow$$
$$9x^2 - 12x + 4$$

Which signs change in a squared difference?

The middle product is negative. The constant is positive because $(-2)^2 = 4$.

3

$$49x^2 - 16$$
$$\downarrow$$
$$(7x)^2 - 4^2$$
$$\downarrow$$
$$(7x - 4)(7x + 4)$$

Why is there no middle term?

The cross-products +28x and −28x cancel.

4

×	x	−3
x	x^2	−3x
5	5x	−15

Collect the products of $(x + 5)(x - 3)$.

The middle terms combine to 2x, giving $x^2 + 2x - 15$.

$(p + q)^2 - (p - q)^2$
4pq.

$(p + q)^2 + (p - q)^2$
$2p^2 + 2q^2$.

CHAPTER PROBLEM

Factorise 12x − 18 using the greatest common numerical factor. Check by expanding.

12x − 18 = 6(2x − 3). Multiplying each term by 6 gives the original expression.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Is 3a + 2b always 5ab?	No. With a = b = 2 the two sides are 10 and 20. Unlike terms cannot be merged that way.
02 Use one identity to calculate 99^2 without long multiplication.	$(100 - 1)^2 = 10{,}000 - 200 + 1 = 9801$.
03 Expand $(x + 1)(x + 4)$. How does its value compare with the original at x = 3?	It gives $x^2 + 5x + 4$, always two less. At x = 3 the results are 28 and 30.
04 Factor $9x^2 - 16$ and verify.	$(3x - 4)(3x + 4)$. Multiplying cancels the linear terms.
05 Explain why $(a + b)^2$ differs from $a^2 + b^2$ at a = 2, b = 3.	25 − 13 = 12, equal to the two missing cross-products 2ab.

$$2(x + 3) = 2x + 6$$
$$\downarrow$$
$$2x + 6 = 2x + 6$$
$$\downarrow$$
$$0 = 0$$

What does this final equality mean?

Every allowed x satisfies the original equation.

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