MATHS FIELD GUIDE · GRADE 8

Compound events need a complete outcome model

OPENING PROBLEM

Two independent fair coins. What is P(exactly one head)?

First	Second
H	H
H	T
T	H
T	T

02 Count paths through two stages
03 Rules and worked cases
04 Two tickets without replacement
05 Count outcomes exactly once
06 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Count paths through two stages

List ordered results or multiply probabilities along a path. State what the first draw changes.

1

First	Second
H	H
H	T
T	H
T	T

Two independent fair coins. What is P(exactly one head)?

Two of four paths, HT and TH. Probability 1/2.

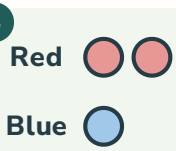
2



Two draws without replacement. What is P(two reds)?

First 2/3, then 1/2. Their product is 1/3.

3



Replace the first counter before drawing again. P(two reds)?

Now both draws have probability 2/3. Multiply to get 4/9.

4

Die 1	Die 2	Sum
1	3	4
2	2	4
3	1	4

Are there three equally likely ways to get four with two independent fair six-sided dice?

Yes. The full ordered grid has 36 cells. Three winning cells give 1/12.

Two fair independent tosses, both heads
1/4.

At least one head
1 − 1/4 = 3/4.

CHAPTER PROBLEM

Why does removing a red counter change the probability of red on the next draw?

It changes both the red count and the total remaining count, from 3/5 to 2/4.

MATHS REFERENCE · GRADE 8

Compound events need a complete outcome model

A compound event combines more than one stage or condition. Independence means information about one event does not change the probability of the other.

A replacement decision can change the outcome model between draws.

Four cases to recognise

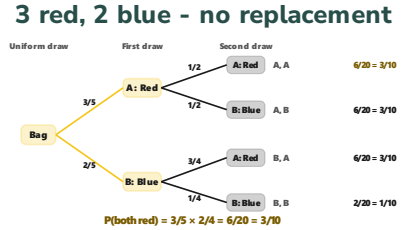
Situation	What to do and why
Two independent fair coins, at least one head	Complement TT has probability 1/4. Answer 3/4.
Three blue and two red, draw two without replacement	Both blue has probability (3/5)(2/4) = 3/10.
Same bag, draw with replacement	Both blue has probability $(3/5)^2 = 9/25$.
One red and one blue without replacement	Count both orders: (2/5)(3/4) + (3/5)(2/4) = 3/5.

Read the model

Build the idea

A bag holds three red and two blue counters. After a counter is drawn and replaced, the second draw again has red probability 3/5.

Without replacement, drawing red first leaves two red and two blue, giving second-red probability 2/4.



Second draw has one fewer counter

Why it works

Multiply probabilities along a specified sequence of conditional stages. With replacement, red then red has probability (3/5)(3/5) = 9/25.

Without replacement it has probability (3/5)(2/4) = 3/10. The changed denominator records the changed total.

independent One event does not alter another's probability.
conditional probability Chance given information about a prior event.
complement All outcomes outside an event.

Heads	Tosses	Share
3	4	3/4
5	10	1/2

Which trial set landed closer to half heads?

The second. The first did not disprove a fair-coin model.

WORKED INVESTIGATION

Two tickets without replacement

A box contains three gold tickets and two silver. Draw two tickets in order without replacing the first. Track how the first draw changes the box.

Look at this part	What it tells you
First gold	3/5
Second gold after gold	2/4
Both gold	3/10

Work it through

Exactly one gold can happen in two orders. Gold then silver gives (3/5)(2/4); silver then gold gives (2/5)(3/4). Add them to get 3/5.

Catch the mistake

Using 3/5 for the second gold ignores the missing ticket. Redraw the remaining contents after each possible first draw.

Compare the cases

Follow an example

For two fair coin tosses, “at least one head” includes HH, HT and TH. Its probability is 3/4.

Its complement is no heads, or TT, with probability 1/4. Subtracting that from one is another way to obtain 3/4.

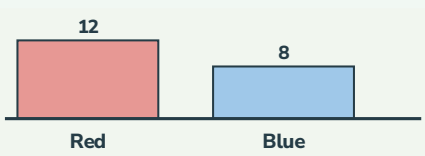
What changes? What stays true?

Mutually exclusive events cannot happen together. This is different from independence. Two independent events can happen together.

A tree or complete list must preserve order when the experiment distinguishes first and second stages.

Change one condition

With replacement, both gold becomes $(3/5)^2 = 9/25$. Replacing the first gold keeps the second gold chance higher.



Estimate the chance of red from these twenty draws.

12/20 = 3/5. Record whether each counter was replaced before comparing trials.

ILLUSTRATED PROBLEM SHEET

Count outcomes exactly once

The counting unit depends on whether order matters and whether an object can be used twice.

1

First	Possible next
A	B or C
B	A or C
C	A or B

Choose a captain then a deputy from three pupils. How many results?

Three choices, then two remaining choices. Six ordered outcomes.

2

AB and BA
↓
Same two-person team
↓
6 ÷ 2 = 3

What if the two roles disappear?

Each unordered pair was counted twice. There are three teams.

3

Die A	Die B
1	5
2	4
3	3
4	2
5	1

How many ordered two-die results give sum six?

Five out of 36 for independent fair dice. The double (3,3) appears once.

4

Tickets 1 to 12
↓
Multiples of 2: 6
↓
Multiples of 3: 4

How many are multiples of 2 or 3?

Subtract the two overlaps, 6 and 12. The union has 6 + 4 − 2 = 8 tickets.

Four pupils in two distinct roles
4 × 3 = 12.

Two-person teams from four
12 ÷ 2 = 6.

CHAPTER PROBLEM

Flip two fair coins independently. Find the probability of exactly one head.

HH, HT, TH and TT are equally likely. Two of the four outcomes have exactly one head, so the probability is 1/2.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Why do HT and TH count separately?	The first and second tosses occupy different positions, even when the number of heads matches.
02 Is an unordered pair of die faces always as likely as any other unordered pair?	No. A non-double has two orders; a double has one.
03 Without replacement, what is the chance of at least one gold?	Both silver has probability (2/5)(1/4) = 1/10. Its complement, at least one gold, has probability 9/10.
04 Four white and one black counter. Draw two without replacement. Chance of two black?	Zero. A first black draw leaves no black counters for the second.
05 With replacement from the same bag, chance of exactly one black in two draws?	BW and WB each have probability (1/5)(4/5). Their sum is 8/25.

Set	Hits	Tries
A	3	5
B	9	15

Combine the evidence.

Twelve hits in twenty tries gives 3/5. Pool counts before dividing.

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