MATHS FIELD GUIDE · GRADE 10

Zeros and coefficients

OPENING PROBLEM

Write the monic polynomial for the second row.

Root 1	Root 2	Sum	Product
2	5	7	10
−2	5	3	−10

02 Recover coefficients from two zeros
03 Rules and worked cases
04 From the givens to the answer
05 Connect factors, roots and the constant term
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Recover coefficients from two zeros

For $ax^2 + bx + c$ with $a \neq 0$, the two zeros (counting repeats) sum to $-b/a$ and multiply to c/a.

1

Root 1	Root 2	Sum	Product
2	5	7	10
−2	5	3	−10

Write the monic polynomial for the second row.

$x^2 - 3x - 10$. The sign of the middle coefficient is the opposite of the sum.

2

Zeros −1 and 4
↓
$(x + 1)(x - 4)$
↓
$x^2 - 3x - 4$

What if the leading coefficient must be 3?

Triple the whole polynomial to obtain $3x^2 - 9x - 12$. Its zeros stay fixed.

3

$2x^2 - 5x + c$
↓
One zero = 2
↓
$8 - 10 + c = 0$

Find c and the second zero.

$c = 2$. The sum is 5/2, so the other zero is 1/2. Their product is 1.

4

Sum = 0
↓
Product = −9
↓
$x^2 - 9$

Can opposite zeros make the linear coefficient vanish?

Yes. The zeros 3 and −3 add to zero.

Roots 0 and 7
monic polynomial $x^2 - 7x$.

Roots −7 and 7
monic polynomial $x^2 - 49$.

CHAPTER PROBLEM

For $2x^2-9x+4$, find the sum of zeros.

4.5. Here $a = 2$ and $b = -9$. The sum is $-b/a = 9/2 = 4.5$.

MATHS REFERENCE · GRADE 10

Zeros and coefficients

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Sum and product	For $3x^2 - 5x - 2$, sum = 5/3 and product = −2/3. Keep the coefficient signs.
Recover a missing zero	If one zero is 2, the other is $5/3 - 2 = -1/3$. Their product checks.
Scaling a polynomial	Multiplying all coefficients by −2 leaves both coefficient ratios and therefore the zeros unchanged.

Read the representations

Expand the factors to read the coefficients

If α,β are the zeros of ax^2+bx+c with $a\neq0$, write $p(x)=a(x-\alpha)(x-\beta)$.

Expanding gives $ax^2-a(\alpha+\beta)x+a\alpha\beta$. Matching coefficients yields $\alpha+\beta=-b/a$ and $\alpha\beta=c/a$.

1. **Factor form** $a(x-\alpha)(x-\beta)$
2. **Expand** $ax^2-a(\alpha+\beta)x+a\alpha\beta$
3. **Compare** Sum = $-b/a$; product = c/a

Keep signs attached to the coefficients

In $2x^2-7x+3$, $a=2, b=-7, c=3$. The sum is 7/2 and product 3/2.

A negative sum does not mean both zeros are negative; the product also matters.

$$6x^2 + x - 2 = 0$$
$$\downarrow$$
$$(3x + 2)(2x - 1) = 0$$
$$\downarrow$$
$$x = -2/3 \text{ or } 1/2$$

Which products make the middle term?

The cross-products are −3x and +4x, leaving +x.

WORKED EXAMPLES

From the givens to the answer

Verify with a nonmonic polynomial

1. $2x^2-7x+3=(2x-1)(x-3)$, so the zeros are 1/2 and 3.
2. Sum: $1/2+3=7/2=-(-7)/2$. Product: $(1/2)\times3=3/2$.
3. Both match the coefficient relationships.

Reconstruct the missing zero

The zeros of $3x^2 - 12x + 9$ add to 4. One is 1. Find the other and check the constant term.

1. The other zero is $4 - 1 = 3$. Their product is $1 \times 3 = 3$, matching $c/a = 9/3$. Factoring gives $3(x - 1)(x - 3)$.

Check another case

Example 1

For $3x^2+5x-24$, find the product of zeros.

-8

Product = c/a.

Example 2

Zeros are 9 and −2. Find the x-coefficient in the monic polynomial.

-7

The x-coefficient is minus the sum of zeros.

What if the roots have opposite signs?

For $2x^2 + x - 6$ the product of the zeros is −3, so one is positive and one negative. Their sum must be −1/2.

Factor as $(2x - 3)(x + 2)$. The zeros 3/2 and −2 satisfy both checks.

$$x^2 - 6x + 5 = 0$$
$$\downarrow$$
$$(x - 3)^2 = 4$$
$$\downarrow$$
$$x = 1 \text{ or } 5$$

Why do two roots appear?

Both 2 and −2 square to 4, so x − 3 can take either sign.

ILLUSTRATED PROBLEM SHEET

Connect factors, roots and the constant term

A product is zero when at least one factor is zero.

1

$$(x - 2)(x + 3)$$
$$\downarrow$$
$$x^2 + x - 6$$

Which inputs give zero?

2 and −3. Each makes a different factor vanish.

2

x	−3	0	2
p(x)	0	−6	0

Which table entry is the vertical intercept?

At x = 0 the output is −6. The two zero outputs locate horizontal intercepts.

3

$$2(x - 2)(x + 3)$$
$$\downarrow$$
$$2x^2 + 2x - 12$$

Does doubling the polynomial change its roots?

No. A nonzero constant factor changes the outputs but preserves the zeros.

4

$$\text{Sum} = -1$$
$$\downarrow$$
$$\text{Product} = -6$$
$$\downarrow$$
$$x^2 + x - 6$$

Reconstruct a monic quadratic from these two root facts.

Use $x^2 - (\text{sum})x + \text{product}$. Its factors are (x − 2) and (x + 3).

$p(x) = (x - 4)^2$
one distinct zero.

$p(x) = (x - 4)(x + 4)$
two distinct zeros.

CHAPTER PROBLEM

For $3x^2+5x-15$, find the product of zeros.

-5. Product = c/a.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 A monic quadratic has zeros that add to 6 and multiply to 9. How many distinct zeros?	$x^2 - 6x + 9 = (x - 3)^2$, so one distinct zero.
02 Can a nonzero quadratic have three distinct real zeros?	No. Three distinct linear factors would require degree at least three.
03 Zeros are 7 and −2. Find the x-coefficient in the monic polynomial.	-5 The x-coefficient is minus the sum of zeros.
04 One zero of $x^2-11x+24$ is 3. Find the other.	8 The zeros sum to 11. The other zero is 11 − 3 = 8. Their product 3 × 8 confirms 24.
05 For ax^2+bx+c, $a\neq0$, which is the sum of zeros? A) b/a B) −b/a C) c/a D) −c/a	−b/a Expansion of $a(x-\alpha)(x-\beta)$ fixes the minus sign.
06 If zeros of x^2-3x+m have product 8, find m.	8 For a monic polynomial the constant is the product.

$$x^2 - 2x - 1 = 0$$
$$\downarrow$$
$$D = 8$$
$$\downarrow$$
$$x = 1 \pm \sqrt{2}$$

When is an exact radical useful?

The discriminant is positive and is not a perfect square. The formula preserves both exact roots.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Derive the sum and product relationships from factor form.	Expand $a(x-\alpha)(x-\beta)$. Compare coefficients of x and the constant with ax^2+bx+c. Divide by nonzero a.
08 Can two real zeros have positive sum and negative product? Explain.	A negative product requires opposite signs. For example 5 and −2 have sum 3 and product −10.
09 For $2x^2-11x+5$, find the sum of zeros.	5.5 For $ax^2 + bx + c$ the sum is $-b/a$. Here $b = -11$ and $a = 2$, giving 11/2. As a check, $(2x - 1)(x - 5)$ has zeros 1/2 and 5.
10 For $3x^2+5x-18$, find the product of zeros.	-6 Divide the constant coefficient by the leading coefficient. The product is $-18/3 = -6$.
11 For $2x^2-15x+7$, find the sum of zeros.	7.5 Use $-b/a$ with $a = 2$ and $b = -15$. The sum is 15/2. The factors $(2x - 1)$ and $(x - 7)$ confirm zeros of 1/2 and 7.

$$w(w + 3) = 40$$
$$\downarrow$$
$$(w - 5)(w + 8) = 0$$
$$\downarrow$$
$$w = 5$$

Which algebraic answer fails the model?

A side length cannot be −8. The rectangle is 5 by 8.

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